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EFFECT OF AXIAL LOAD ON THE SHEAR STRENGTH OF REINFORCED CONCRETE BEAMS

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I. INTRODUCTION

1. Object and Scope

The object of this investigation was to study, by means of tests, the effect of axial load on the behavior of reinforced concrete beams failing in shear.

The investigation consisted of tests on 20 beams and the analysis and interpretation of the results. The variables included axial load, span, steel percentage, and unintentional differences in concrete strength. The beams were all simply supported and loaded at midspan through an integrally cast column stub. The ultimate application of the results of this investigation is to the design of members in reinforced concrete box culverts. These tests are those designated as Series A.2.3.1 in "A Suggested Program of Tests for the Development of Criteria for the Structural Design of Reinforced Concrete Box Culverts," Ref. 1.

This report deals exclusively with reinforced concrete beams without web reinforcement.

2. Acknowledgments

This investigation was conducted by the Structural Research Laboratory in the Engineering Experiment Station of the University of Illinois, in cooperation with Ohio River Division Laboratories, Corps of Engineers, U. S. Army under Contract DA-33-017-CIVENG-56-6.

The general direction of the work was provided by Dr. N. M.

Newmark, Research Professor of Structural Engineering. The immediate supervision of the program was provided by Dr. C. P. Siess, Research Professor of Civil Engineering.

Appreciation is expressed to S. R. Bernaert and R. D. DeCossio, Research Assistants in Civil Engineering for their aid in conducting the tests and in preparing this report. Mr. Wyck McKenzie, Junior Laboratory Mechanic, was very helpful in conducting the tests and in fabricating specimens and equipment.

This report was written as a thesis under the direction of Dr. Siess and his helpful comments are gratefully acknowledged.

3. Notation

The following notation has been used in this report:

Distances

a = length of the shear span (see Fig. 1)

b = width of beam

d = effective depth of reinforcement

h = overall height of beam

kd = depth of compression zone of concrete as determined by "straight-line" theory

$k_u d$ = depth of compression zone

L = length of span between centers of supports

x_x = horizontal distance from center of support to point at which crack causing failure intersected the reinforcement

y = midspan deflection

Forces

C = total internal compressive force in concrete

N = axial load

P_c = total load on beam at diagonal tension cracking

P_u = ultimate load (corresponding to failure of beam)

V = total shearing force

V_c = total shearing force at diagonal tension cracking

V_u = total shearing force at ultimate

Moments

M_f = theoretical ultimate flexural moment

M_s = theoretical limiting shear moment

M_u = measured ultimate bending moment
$= V_u a$

Stresses

f'_c = compressive strength of concrete determined from 6 by 12-in. control cylinders

f_r = modulus of rupture of concrete determined from 6 by 6 by 20-in. control beams

f_s = stress in tensile reinforcement

f_y = yield strength of reinforcement

v_c = nominal unit shearing stress at diagonal tension cracking

$$= \frac{V_c}{\frac{7}{8}\,bd}$$

Strains

e_s = unit strain in the reinforcement

e_u = limiting strain in concrete

Constants, Parameters, and Ratios

A_s = total area of reinforcement

a/d = shear span-depth ratio

E_s = modulus of elasticity of steel

k_1 = ratio of average compressive stress to maximum compressive stress in the concrete stress block

k_2 = distance from top of beam to line of action of compressive force C, divided by $k_u d$

k_3 = ratio of maximum compressive stress in concrete stress block to cylinder strength, f'_c

p = percentage of steel
= A_s/bd

II. EXPERIMENTAL PROGRAM

4. Description of Specimens

All the beams tested were rectangular in cross-section, and reinforced in tension only. The properties of the beams are given in Table 1.

The nominal concrete strength was 4000 psi; however, the actual strengths varied somewhat from this value. The other variables were span length and percentage of reinforcement.

A typical beam is shown in Fig. 1. All beams had an unreinforced column stub cast integrally at midspan, as shown in Fig. 1. On all beams but A-2, an external stirrup was placed at each end outside the supports to prevent failure by splitting at the level of reinforcement. The reinforcement extended to one inch from the end of the beam.

The length of the shear span, a, varied from 20 to 60 in. in 10 in. intervals. Four beams of each length were tested. Beams A-1 through A-5 and beams B-1 through B-5 were all reinforced with 3 No. 4 bars. Beams A-11 through A-15 and beams B-11 through B-15 were all reinforced with 2 No. 9 bars. The beams designated A were all tested with no axial load. The beams designated B were all tested with an axial load of 20 kips.

5. Materials

(a) Cement. Two brands of cement were used. Beams B-1 through B-5 and A-11 were made with Atlas Type I cement. All other beams were made with Marquette Type I cement.

(b) Aggregate. Wabash River sand and gravel were used in all beams. The maximum size of the coarse aggregate was about one inch, with a fineness modulus of 6.5 to 7.0 and contained a rather high percentage of fines. The fineness modulus of the sand varied between 2.7 and 3.2. Both aggregates have passed the usual specifications. The absorption was about one per cent by weight. The aggregate was purchased from a local dealer.

(c) Concrete Mix. One basic mix was used in an attempt to obtain the same concrete strength in all test specimens. The actual proportions and properties of the concrete mixes are given in Table 2.

(d) Reinforcing Steel. The reinforcing steel was purchased in two lots. One 2-ft long coupon was cut from each bar and tested upon arrival.

The No. 4 bars used were all high grade deformed bars. The yield strength varied from 66,500 to 68,000 psi. The ultimate strength averaged 121,000 psi. The average modulus of elasticity for these bars was 27,400,000 psi.

The No. 9 bars were all intermediate grade deformed bars with yield points from 45,500 to 57,000 psi. The average ultimate strength was 78,800 psi. The average modulus of elasticity was 28,000,000 psi.

The values of the average yield point strength for the bars used in each beam are given in Table 1. The bars used in each beam were matched according to their yield points, using bars cut from the same piece in one beam whenever possible.

6. Fabrication and Curing of Specimens

Before the reinforcement was assembled for beams with No. 9 bars, 6-inch gage lines for mechanical strain gages were marked on one outside bar, and the gage holes punched and drilled. Corks of 1 3/8 in. diameter were wired to the bars over the gage holes in order to form core holes in the side of the beam to provide access to the gage holes.

All beams were cast in a steel form with adjustable end plates. The reinforcing steel was held in place by 2 or 3 chairs made of 1/4 in. mild steel bars. Two or three pieces of 3/4 in. pipe, acting as spacers, were distributed along the beam. To facilitate handling, a 1/4 in. steel hook was embedded in the concrete at each end of the beam.

All concrete was mixed from three to eight minutes in a non-tilting drum-type mixer of 6-cu ft capacity. A butter mix was used prior to the mixing of the first batch. Two batches of concrete were used for each beam. The first batch was placed in a horizontal layer along the bottom of the beam.

Four to six 6 by 12-in. control cylinders and one 6 by 6 by 20-in. control beam were cast from each batch. The concrete was placed in the forms and cylinder molds with the aid of a high-frequency internal vibrator.

Several hours after casting, the top surface of the beam was trowelled smooth, and the cylinders capped with neat cement paste. The beams and control cylinders were removed from the forms one day after casting and placed in a moist room for six days. They were then stored in the laboratory until testing.

7. Testing Equipment

A typical test setup for the beams of Series A, which were tested without axial load, is shown in Fig. 2. The beams of Series B were tested with an axial load of 20 kips by means of the equipment shown in Figs. 3 and 4.

(a) Lateral Loading Equipment. The lateral load was applied by four Blackhawk hydraulic jacks of 10-ton capacity each. The jacks reacted against a steel beam attached to a frame which was anchored to the laboratory floor. The jacks were connected by high pressure hose to a brass manifold, which in turn was connected to a measuring gage and a hydraulic pump. The jacks were held with their bases against the reaction beam by two 1 by 1 by 1/8-in. angles clamped to the reaction beam.

The load was transmitted from the jack rams to the beam through 1.5-in. diameter chrome steel alloy balls. The balls rested in 1/8-in. depressions in the end of the ram and in the loading block.

The loading block was a 6 by 12 by 2-in. steel plate, with depressions formed on its upper surface to receive the steel loading balls. These depressions were on the centerline in the long direction and spaced at 3 in. as shown in Fig. 1. The support bearing blocks were 6 by 6 by 2-in. steel plates. The loading block and the bearing blocks were set in plaster.

The support block on one end rested on a 4-in. diameter half round, the other on a 2-in. diameter roller. The roller and half round both were supported by 6 by 12 by 2-in. steel plates seated in plaster on concrete abutments.

Two pressure measuring gages were used in the loading system, a 5,000 psi gage for small loads and a 10,000 psi gage for large loads. The area of each jack ram was approximately 2 sq in; thus the capacity of the system was 80,000 lb. Prior to their use, both gages were calibrated with the four jacks in a testing machine; thus the total load was read directly during testing operations. It was estimated that the accuracy of the system was $\pm$ 0.2 kips with the 5000 psi gage, and $\pm$ 0.5 kips with the 10,000 psi gage. The latter was used only where the total load exceeded 45 kips.

(b) Axial Loading Equipment. The axial loading equipment was a completely separate unit, as seen in Fig. 3. It consisted of a hydraulic jack operating against one end of the beam, with the reaction to the jack supplied by tension rods acting against the other end of the beam. To allow the ends of the beam to rotate, a half round rocker was included at each end. These rockers were welded to

12 by 6 by 2-in. plates which transmitted the load to the ends of the beam through leather pads. The jack operated on one end between two 10 by 12 by 2-in. plates, which were countersunk to receive the ram and the base of the jack. Four 7/8-in. diameter threaded steel rods were used to connect the jack bearing plate to the plate acting on the other end of the beam. Two rods passed on each side of the beam. The rods were threaded so that the system could be adjusted to accommodate beams of various lengths.

A simplex hydraulic jack of 30 ton capacity was used. It was connected by means of a hose to a 5000 psi gage and a pump. The gage was calibrated with the jack in a testing machine before the equipment was used. It was estimated that the load was measured within 0.2 kips.

Care was taken in assembling the equipment that the center-line of the jack and the half round were at midheight of the beam before the load was applied. The equipment was supported on the beam by friction. No difficulties were encountered in this respect.

(c) Deflection Apparatus. Deflections were measured at midspan on all beams, and at the quarter points of the span on all but the 52-in. beams. Dial indicators reading to 0.001 in. were used under the beam to measure deflections. The dials were supported by posts attached to a deflection frame as shown in Fig. 2. The frame was a 2 1/2 by 2 1/2 by 3/4-in. angle cemented to the support blocks with plaster of paris.

(d) Strain Measurements. Strains were measured in the steel only on the beams with 2 No. 9 reinforcing bars; that is, beams A-11 through A-15 and B-11 through B-15. A Berry-type mechanical gage with a sensitivity of 0.00003 in. per in. was used on six inch gage lengths. Strains were measured on one side only. The locations of the gage lines are shown on Fig. 1. The number of gage lines depended on the length of the beam, and varied from 7 to 21.

8. Testing Procedure

Once the beams to be tested with axial load were in the testing frame, and the axial load applied, there was little difference in the testing procedure from that for beams with no axial load. The axial load was checked regularly during the test, and maintained at 20 kips by adjusting the pressure when necessary. The remainder of the testing procedure outlined below applies to beams both with and without axial load.

The lateral load was applied in 10 to 15 approximately equal increments up to failure. After each increment of loading, the valve between the pump and the jacks was closed. Deflection readings were then recorded, and cracks were observed and marked with ink. There was usually some drop-off in the load and some increase in deflection while the cracks were being marked. These changes were noted before the next load increment was applied. The average length of test was about 6 hours.

Photographs were taken of the beams at important stages in the crack development, and after failure.

For the beams with 2 No. 9 reinforcing bars, several measurements of strain in the tensile steel were taken during the course of the test.

The location and height of the crack that led to failure were measured.

The concrete control cylinders and flexural control beams were tested on the same day that the beam was tested.

III. TEST RESULTS

9. Test Data

The results of the tests are summarized in Tables 3 and 4, and Figs. 5 through 9.

Table 3 gives the location and height of the crack which led to failure. There was some difference between the values measured on either side of the beam, probably due to non-symmetrical loading and to lack of homogeneity in the concrete. The values given in Table 3 are average values.

Table 4 includes the diagonal tension cracking load P_c, the untimate load P_u, the ultimate moment M_u, the mode of failure, the average concrete strength, and the maximum steel stress for the beams with two No. 9 bars. The values of loads and moments are for the added load only; the dead load was generally less than 2 per cent of the total and was therefore neglected.

The diagonal tension cracking load was determined by observation. It is that load at which a definite diagonal tension crack was first observed; that is, when it became apparent that a particular crack was assuming major importance. The cracking load for the longer beams was usually associated with a sudden development of the diagonal tension crack, and thus was well defined. For the shorter beams the crack development was much more gradual; consequently, the cracking load was not so clearly defined, and in fact involved considerable vagueness.

The ultimate load P_u is the maximum load that the beam carried. In almost all cases P_u was also the load at which the beam collapsed. In a few cases the beam did not completely fail at P_u, but at a load slightly less; however, P_u is considered the failure load since, if it had remained on the beam for a short time, it would have resulted in complete failure.

The ultimate moment M_u is the moment at the face of the column stub at the load P_u.

Three modes of failure were observed; shear-compression, diagonal tension, and flexure, as indicated in Table 4 by the symbols S, DT, and F respectively.

A shear-compression failure is defined as failure by destruction of the compression zone above a diagonal tension crack at a load greater than the cracking load.

Diagonal tension failures are defined as failures at the cracking load.

Flexural failure for these beams which were under-reinforced, is defined as failure by crushing of the concrete in the compression zone after the reinforcement has yielded but before the development of diagonal tension cracks.

The concrete strength given in Table 4 is the average value of Batch 2 which was placed in the top of the beam.

The steel stresses in Table 4 are the results of the measurements taken of the strains in the tension reinforcement. They are the maximum steel stresses; obtained by extrapolating to the ultimate load. For each beam the maximum steel stress was at or near the column face section. It should be pointed out that since the values given are extrapolated they may involve some error.

The load-deflection curves for all the beams are given in Figs. 5 and 6.

Figures 7, 8 and 9 show typical results of the tensile steel strain measurements. Figs. 7 and 8 show the results for beams B-11 and B-13, which failed by shear-compression. Figure 9 is for beam A-15, and is representative of the results for diagonal tension failures.

10. Behavior Under Load

In the early stages of testing the beams all behaved similarly. The behavior in the later stages depended on the extent of diagonal tension cracking and the mode of failure.

Until the appearance of diagonal tension cracks, the beams all behaved in the usual manner of concrete beams in flexure. The first cracks appeared at midspan, and extended vertically, increasing in height with increasing load. With increased load, additional cracks developed in the shear span in the region near the support. These cracks were usually somewhat inclined from the beginning in agreement with the pattern of principal stress. The deflections in the early stages of loading were nearly proportional to the load as can be seen in Figs. 5 and 6. As the load became larger, greater deflections were required to develop additional resistance.

The steel stresses for loads below the diagonal tension cracking load were very nearly proportional to the moment as can be seen from Figs. 7, 8 and 9. They were equal to zero at the support and increased linearly toward midspan.

At this point it is necessary to discuss behavior according to the mode of failure.

(a) Shear-Compression Failures. Figure 10 shows the crack development for a typical shear failure. At a load of 22.0 kips the cracks were primarily flexural. Fig. 10(b) shows that at the cracking load of 29.6 kips very extensive diagonal cracking had occurred but the beam was still capable of carrying increased load. The beam is shown after failure in Fig. 10(c). All of the beams which failed in shear-compression behaved very much like Beam B-12 in Fig. 10. The following observations may be made as being characteristic of the shear-compression failures:

(1) Diagonal tension cracks developed before failure. The reader is again referred to the load-deflection curves in Figs. 5 and 6. For those beams which failed in shear-compression, there are sharp breaks in the curve. These breaks were caused by the formation of diagonal tension cracks, without loss of load carrying capacity. The very shortest beams do not demonstrate these breaks because the diagonal tension cracks developed gradually. Beam B-3 failed after only one end had developed a diagonal tension crack, and thus was actually a transition failure between shear-compression and diagonal

tension failure. Beam B-13 failed just after the development of the second diagonal tension crack, and thus was also very nearly a diagonal tension failure.

(2) The extent to which the beam carried increased load after diagonal tension cracking depended on the a/d ratio. This can be seen in Figures 15 and 16 which show the shear at diagonal tension cracking and at ultimate for all beams except A-5 and B-5 which failed in flexure. It can be seen that the increased strength after diagonal tension cracking was very large for the smallest a/d ratio but decreased rapidly with increased a/d.

(3) Final failure occurred by destruction of the compression zone above a diagonal tension crack. The concrete did not crush in the same manner as in flexural failures, however. The failure was usually on a plane of 45° or less to the vertical, and the break was clean and sudden.

(4) Considerable cracking occurred along the steel before failure, extending from the point where the diagonal tension crack intersected the steel toward the support. At failure the steel was always completely separated from the concrete above it for at least half the length of the span.

(5) Diagonal tension cracking caused a very marked change in the steel stress distribution, as can be seen in Figs. 7 and 8. After cracking, the steel stress was almost constant over the whole span. It should be noted here that the cracking load for B-11, as given in Table 4, is 37.8 kips, at which load there was already

considerable stress redistribution as can be seen in Fig. 7. This difference is because of the gradual crack development, and was present in all of the 52-in. beams, but not in the longer ones, since for the latter the crack development was sudden. This is illustrated in Fig. 8 for beam B-13. For this beam, the load increment at which diagonal tension cracks formed on each end can easily be inferred from the marked changes in the stress distribution.

With such high steel stresses near the support after diagonal tension cracking, it is obvious that the bond and horizontal shear stresses near the support, and probably even in the overhang, must be very great in order to develop the steel stress in a few inches. Although external stirrups were used to prevent failure by splitting along the steel at the support, it was never certain how effective they were, since that part of the beam usually suffered some damage during final collapse. It is possible that failure near the support could have triggered the final failure in some instances.

Beam A-2 which failed in diagonal tension did not have external stirrups. It is likely that it would have failed in shear-compression if stirrups had been used. The ultimate load would not have been much higher however, as indicated by the curves of Fig. 15.

(6) All the shear-compression failures occurred at relatively small deflections. However, relatively large increments of deflection were associated with diagonal tension cracking, as can be seen in Figs. 5 and 6.

(b) Diagonal Tension Failures. Ten of the beams tested failed in diagonal tension. Diagonal tension failures are failures at the cracking load. The development of a typical diagonal tension failure is illustrated in Fig. 11 for beam A-14. Until diagonal tension cracking and failure, the beams developed cracks gradually as shown in Fig. 11(a) and (b). When the load reached the diagonal tension cracking load, a long sweeping crack developed, and the beam collapsed, as illustrated in Fig. 11(c). The following observations may be made about the diagonal tension failures:

(1) Failure occurred at the diagonal tension cracking load. This is by definition. It is to be noted however that only one diagonal tension crack occurred for these beams, while in the case of the shear-compression failures both ends developed diagonal tension cracks. (Beam B-3 developed only one crack, but, as has already been noted, it was a transition failure, and thus had some characteristics of a diagonal tension failure).

(2) Final failure occurred by destruction of the compression zone above the diagonal tension crack, and by splitting along the steel from the point where the crack started to the support. In four of the beams, the destruction of the compression zone was similar to that for the shear-compression failures. For the others, there was no evidence of crushing or shearing off. For beams A-3, A-13, and A-14, the compression zone was very slender as can be seen for beam A-14 in Fig. 11(c), and A-13 in Fig. 14(a). It appeared as

though the compression zone buckled. For the beams A-15, B-14, and B-15, the compression zone was not slender, and didn't buckle, but broke under the stub very much like beam B-13 shown in Fig. 14(b). For the last three beams at least it did not appear that the destruction of the compression zone was caused by excessive compressive stresses.

In all cases a horizontal crack developed during failure along the steel from the point where the diagonal tension crack started to the support. It was impossible to determine whether this crack developed before or after the destruction of the compression zone. In the discussion of the shear-compression failures it was noted that there was much cracking along the steel after diagonal tension cracking but before destruction of the compression zone. It was noted also that the steel stress was almost constant over the whole span after diagonal tension cracking. These observations indicate that the steel can be split from the concrete above it before the destruction of the compression zone, and could therefore trigger the failure. For the three beams which failed like beam B-13 in Fig. 14(b) this appeared to be what happened.

(3) The ultimate load was less for the longer beams than for the shorter ones. This is illustrated in Figs. 15 and 16. The diagonal tension failures are those for which the ultimate shear was equal to the cracking shear. The difference in strength between short beams and long beams can be seen to be much less for diagonal tension failures than for shear failures. Beams A-3 and A-13 fall

somewhat low, and beams A-2 and A-12 somewhat high. The analysis of the next chapter indicates that except for beam A-13 these differences are explained by the variation in the concrete strength.

(4) The steel stress distribution up to diagonal tension failure varied almost directly with the moment as illustrated for beam A-15 in Fig. 9.

(5) The deflections at failure were small.

(c) Flexural Failures. Two beams, A-5 and B-5, failed in flexure. They are shown after failure in Fig. 12. Although inclined cracks developed at final failure, these two beams are classed as flexural failures because very large deflections were observed (see Fig. 5), the steel had evidently yielded to permit these deflections, and the concrete adjacent to the stub had begun to crush before the inclined cracks developed. Until final failure, the crack development was gradual. After the reinforcement began to yield, at a deflection of about 0.6 in. for both beams, there was very little increase in load-carrying capacity. As the deformation was increased, horizontal cracks began to develop about 1 1/2 in. under the stub. By the time the concrete began to crush, these cracks had developed for the full length of the stub (see Fig. 12). After some crushing had occurred, inclined cracks developed. The final destruction of the beam was due partly to these inclined cracks.

(d) Effect of Axial Load. Except for small differences in concrete strength, the beams tested with and without axial load were identical. Any differences in behavior or load-carrying capacity should therefore be completely attributable to the 20-kip axial load. The axial load had the following effects:

(1) It changed the pattern of crack development. Flexural cracking was suppressed and the first flexure cracks appeared at a higher load. There was a reduction in the total number of cracks, and also in the height to which they rose. This is illustrated in Fig. 12.

In general, diagonal tension cracks began farther from the support and did not rise as high for beams with axial load as for those without. This is shown in Figs. 13 and 14. Examination of Table 3 shows that the diagonal tension cracks did not start farther from the support in every case for the beams with axial load; however, the depth of the compression zone was greater in every case.

(2) The axial load affected the deformation characteristics. The extent of this effect was much greater for the beams with three No. 4 bars than for the beams with two No. 9 bars, as can be seen in Figures 5 and 6. The axial load raised the load-deflection curves considerably for the beams with the small steel percentage, but only very slightly for those with the high steel percentage. The reason for this difference is explained in a later section. The axial load tended to increase for the beams with three No. 4 bars and had to be adjusted.

(3) The axial load generally raised the diagonal tension cracking load. This effect can be seen in Table 4 and Figs. 15 and 16. The effect was greater for the shorter beams and also for the beams with three No. 4 bars. For beams B-14 and B-15, the diagonal tension cracking load was not raised, but rather was slightly lower than for A-14 and A-15 which had no axial load.

(4) The ultimate load carrying capacity was increased for all beams except B-14 and B-15. The ultimate capacity was affected in much the same way as the diagonal tension cracking load, that is more for the short beams than the long ones, and more for the beams with the low steel percentage. The reader is again referred to Table 4 and Figs. 15 and 16. Beam B-5 which failed in flexure withstood only slightly more load than the corresponding beam without axial load.

(5) The axial load changed the mode of failure from diagonal tension to shear-compression for some of the beams. The shear-compression strength was increased more than the diagonal tension strength and consequently the transition between shear compression and diagonal tension failures was shifted to larger a/d ratios as a result of the presence of axial load. Beams B-2, B-3, B-12 and B-13 failed at loads greater than the cracking load, while the corresponding beams without axial load failed at the cracking load. Beams B-3 and B-13 were not true shear-compression failures however, since B-3 developed only one crack and B-13 did not fail by destruction of the compression zone in the manner of the other shear compression failures. These two

beams were transition failures.

(6) The effect of the axial load on the maximum steel stress at failure was not consistent. For all the beams but B-12 however, the axial load caused the beam to develop greater steel stresses at failure. For beam B-12 the maximum steel stress was 19.7 ksi, while for beam A-12 it was 20.7 ksi.

IV. ANALYSIS OF TEST RESULTS

11. Load-Deflection Characteristics

It has been noted in Section 10(d) that one of the effects of axial load was to alter the load-deflection relationships. For the beams with a small steel percentage, the effect was quite large, while for those with a high steel percentage, it was small.

For a homogeneous elastic beam, the addition of axial load would not affect the deflections due to lateral loading. Reinforced concrete beams, however, develop tension cracks and at high stresses the concrete is not elastic; consequently, the axial load does have an effect on deflection. The extent of the effect depends on the height of the neutral axis and the stress level in the concrete.

For low percentages of steel, the neutral axis lies above the line of action of the axial load. The axial load thus provides a counter moment which tends to reduce the deflections. For high percentages of steel, the neutral axis is lower; consequently, the effect of axial load on deflection is much less.

The addition of axial load also tends to increase the deflections in both cases by increasing the compressive stresses in the concrete and thus causing the onset of inelastic behavior in bending to occur at a lower lateral load.

Figure 17 shows the theoretical effect of a 20-kip axial load on the moment-rotation relationships for the beams tested. Figure 17(a) is for the beams with a small steel percentage, A-1 through A-5 and B-1 through B-5. Figure 17(b) is for beams A-11 through A-15 and B-11 through B-15 which had a large steel percentage.

The curves of Fig. 17 were computed on the basis of a completely cracked section, using the concrete stress block presented in Fig. 4, Ref. 2. In the analysis, the axial load was applied at mid-height of the section, as in the tests.

It can be seen that the axial load raised considerably the curve for the section with a low steel percentage, but only slightly for the section with a high steel percentage. It is believed that these computed relationships account satisfactorily for the difference in the effect of the axial load on the load-deflection curves in Figs. 5 and 6.

12. Diagonal Tension Cracking Load

(a) Beams Without Axial Load. It has been noted that diagonal tension cracks formed after a considerable number of vertical and inclined cracks had been observed (see Fig. 11). Each diagonal tension crack was a continuation of one of the inclined cracks. Since the inclined cracks extended almost to the neutral axis, the diagonal tension cracks extended into the compression zone. The flatness of the diagonal tension cracks and their sudden development are further indications that they occurred in the compression zone.

Diagonal tension cracks occur when the principal tensile stress reaches the tensile strength of the concrete. The variables which affect the principal stress in the compression zone therefore determine the diagonal tension strength of the beam. The stresses in the compression zone are determined by the depth to the neutral axis, the moment at the section, and the shear. Since there is a complex interrelation of the variables involved, and since the pertinent properties of the concrete are not easily evaluated, an accurate and useful mathematical prediction of diagonal tension cracking is neither possible nor practical. The best solution appears to involve a simplified empirical approach which takes into account fairly well the effects of the several variables.

Since this test program did not include concrete strength as a variable, and since only two steel percentages were used, the effect of these two variables was determined from other studies. Bernaert (Ref. 3) found that the nominal unit end shearing stress for uniformly loaded beams at diagonal tension cracking was predicted quite well by the equation:

$$v_c = \frac{120\ p + 4.3}{\frac{L}{d} + 10} \cdot \frac{f'_c}{1 + \frac{0.85\ f'_c}{1000}} \qquad (1)$$

Equation (1) gives values which are too high for the beams reported here, the difference being due to the manner of loading which results in a different shear distribution. For beams with one or two symmetrically placed loads, the length effect is considered more conveniently by the a/d ratio, which has significance in both cases.

In Fig. 18 the shear span-depth ratio is plotted against the quantity $\frac{120\ p + 4.3}{v_c} \cdot \frac{f'_c}{1 + \frac{0.85\ f'_c}{1000}}$ which was computed from the test results. The points on this plot can be fitted with the line 23 + 2 a/d. The nominal unit shearing stress at diagonal tension cracking for beams without axial load is then given by the equation:

$$v_c = \frac{120\ p + 4.3}{23 + 2\ a/d} \quad \frac{f'_c}{1 + \frac{0.85\ f'_c}{1000}} \tag{2}$$

The correlation between the test values for each beam, and the values computed by equation (2) is shown in Table 5(a). The correlation is very good, except for beam A-13, for which the error is 13 per cent.

The data from tests on beams under two concentrated loads reported by Feldman and Siess in Ref. 4 are also plotted on Fig. 18. The maximum error for these beams by equation (2) was found to be 7 per cent. Equation (2) is therefore as good for two symmetrically placed loads as for centerline loading through a stub. The stub apparently does not affect the cracking load.

(b) Beams With Axial Load. Except for two beams, the axial load increased the diagonal tension cracking load. The amount of the increase depended on the length of the beam and the steel percentage. Increasing the length and increasing the steel percentage both decreased the effect of the axial load.

The axial load has two effects. It lowers the neutral axis and it increases the compressive stresses, both of which decrease the principal tensile stresses. In the discussion of deflections, however, it was seen that for high values of steel percentage the neutral axis was shifted very little. Also, for high values of moment, associated with longer beams, the concrete in the compression zone becomes inelastic and consequently the resistance to shear is reduced. An expression which gives the increase in strength due to axial load must therefore decrease with increasing steel percentage and increasing length.

For a homogeneous elastic beam, the effect of axial load on the diagonal tension strength is simply an addition to the strength without axial load. For reinforced concrete, the effect had to be modified to decrease with p and a/d for the reasons stated.

The equation for nominal unit shearing stress at diagonal tension cracking was written in the form:

$$v_c = \frac{120\ p + 4.3}{23 + 2\ a/d} \quad \frac{f'_c}{1 + \frac{0.85\ f'_c}{1000}} + \frac{N}{\frac{7}{8}\ bd}\ (K_1 - K_2 p - K_3\ a/d)$$

The first term in this equation is from equation (2) while the second term represents the additional strength due to the axial load. The linear form of the effect of p and a/d on the axial load contribution was chosen because not enough tests were made to warrant a more complicated relation. The constants K_1, K_2, and K_3 were determined empirically from the test results for the beams with axial load, and the resulting equation for nominal unit shearing stress at diagonal tension cracking is:

$$v_c = \frac{120\ p + 4.3}{23 + 2\ a/d} \quad \frac{f'_c}{1 + \dfrac{0.85\ f'_c}{1000}} \ + \frac{N}{\frac{7}{8}\ bd} (0.270 - 3.4p - 0.034\ a/d) \qquad (3)$$

The correlation between the test values for each beam, and the values computed by equation (3) is shown in Table 5(b). Except for beams B-1 and B-11 with the shortest spans, the agreement is very good. It has been stated that the cracks developed slowly for the shortest beams and that there was already considerable stress redistribution at the observed cracking load. This probably is the reason for the difference between the observed and computed values for these beams.

Equation (3) is equally good for beams with and without axial load, since for beams without axial load N = 0, and the equation reduces to equation (2).

13. Shear-Compression Failures

A shear-compression failure has been defined as failure by destruction of the compression zone above an inclined crack at a load greater than the diagonal tension cracking load. Eight of the beams tested failed in this manner; two without axial load and six with axial load. Only the shortest beams of those tested without axial load failed by shear-compression, but with the addition of axial load the two next longest beams for both steel percentages also failed in this manner. It has been pointed out that beams B-3 and B-13 are considered transition failures.

Since the number of beams failing in shear-compression is small, the analysis consists only of comparing the test results with what has previously been postulated about this mode of failure.

(a) Beams Without Axial Load. Recent investigators have tried to correlate this type of failure on the basis of a limiting moment. Laupa (Ref. 2) gives the following equation for the limiting shear moment for a beam without axial load:

$$M_s = bd^2 f'_c k \left(0.57 - \frac{4.5 f'_c}{10^5}\right) \tag{4}$$

Figure 19 shows the ultimate moment plotted against the shear span-depth ratio. It includes all of the beams with and without axial load. The beams designated "S" in the legend are those which failed by shear-compression. The dotted line is the value of M_s by equation (4), using the concrete strength of the beams without axial load. Equation (4) includes only properties of the section, and would be a horizontal line except for the variations in concrete strength.

Beams A-1 and A-11 were the only two beams without axial load that failed by shear-compression. Their ultimate moments as given in Fig. 19 are seen to be in reasonable agreement with the predictions of equation (4).

Beams A-2 and A-12 failed in diagonal tension and, according to the theory of shear failure presented in Ref. 4, should have failed at a moment greater than M_s. It can be seen in Fig. 19 that the measured ultimate moment for these two beams was actually less than the computed value of M_s. The difference between the measured and computed values is 11 per cent for beam A-2 and 14 per cent for beam A-12. These variations however, are within the range of accuracy of $\pm$ 15 per cent which has previously been assigned to equation (4) (Ref. 2).

Beams A-3 and A-13 also failed in diagonal tension and because of their greater a/d values should have failed at ultimate moments significantly in excess of the computed shear-compression moment M_s from equation (4). From Fig. 19 it is seen that this condition is satisfied fairly well for beam A-3, but rather poorly for beam A-13; whereas the measured ultimate moment exceeded the computed value for beam A-3 by 55 in-kips, they were nearly equal for beam A-13.

So far as this investigation indicates, it can be said that equation (4) is, within the assigned accuracy, a fair measure of the shear-compression strength of beams without axial load.

It is interesting to note the shapes of the curves in Fig. 19. All four curves rise at the small a/d value of 2, and beam B-1 at this value of a/d developed a moment larger than the flexural ultimate. The values of ultimate shear for the shortest beams in Figs. 15 and 16 are clearly not of the same order of magnitude as the others. It has been noted that these beams did not develop diagonal tension cracks suddenly or of the same shape as those in longer beams, but had rather steep cracks which did not flatter out, and which developed slowly. Considering these facts, it appears that the behavior of short beams is different than that of longer beams, perhaps fundamentally.

(b) Beams With Axial Load. The three shortest beams for each steel percentage tested with axial load failed by shear-compression, as indicated in Fig. 19.

It has been suggested that the addition of axial load does not materially increase the strength of a beam against shear-compression failures, and may even decrease it. In Ref. 2, an extrapolation of Laupa's empirical expression on a semi-rational basis indicated that the compressive force was increased with axial load, but the increase was almost all or more than used up in counteracting the moment of the axial load; consequently, the lateral moment was almost unchanged. The curves of Fig. 19 indicate that for the beams of this investigation, there was a significant increase in strength for the beams with axial load. It has been stated earlier that the shear-compression strength was increased more than the diagonal tension strength so that beams B-2, B-3, B-12 and B-13 failed by shear-compression while the corresponding beams without axial load failed by diagonal tension.

The presence of axial load not only increased the total compressive force in the concrete but also the total tension force in the steel. This is indicated by the steel stresses recorded in Table 4 where, except for beam B-12, the maximum steel stresses were all higher for the beams with axial load than for the corresponding beams without axial load. For beam B-12 the maximum stress was 1.0 ksi less than for beam A-12. That the steel stresses were necessarily larger for the beams with axial load can easily be shown. Taking moments about the center of compression at the point of maximum moment at failure yields the following equation:

$$A_s f_s\ (d - k_2 k_u d) = V_u a - N\ (\frac{h}{2} - y - k_2 k_u d) \qquad (5)$$

Equation (5) is solved for f_s in Table 6 using the measured values for $k_u d$, and $k_2 = 0.50$. The actual value of k_2 was probably something different from 0.50; however, since the actual value is not known, and since its effect is small, it was considered suitable to assume 0.50 for purposes of comparison.

There are several things to be noticed in Table 6. First, the moment of the steel stress at failure (column 6) was higher for beams with axial load than for beams without axial load. The amount of the increase ranged from 26 in-kips for beam B-12 to 92 in-kips for beam B-11. Second, there was no significant difference in the amount of the increase for the beams with the high and the low steel percentages. The increase was greatest for the shortest beams and less for the longer ones. This cannot be interpreted to have significance, however, since the longer beams without axial load failed in diagonal tension and the moment at failure was presumably greater than the shear moment. Third, the computed steel stresses at failure compare fairly well with the measured values, well enough at least to confirm the trends which are being demonstrated. The larger differences for some of the beams is doubtless due to the extrapolation used to determine the values at ultimate.

This analysis leads to one important conclusion. It has been noted that both the compressive force in the concrete and the tension force in the steel were larger at failure for the beams with axial load than for the corresponding beams without axial load. These conditions cannot be satisfied by any theory which uses as criteria a single limiting strain in the concrete and any ordinary strain distribution whether it be a straight line or one modified by a concentration factor which does not depend on the axial load. For if the strain in the steel is determined by the position of the neutral axis and a limiting concrete strain, then the steel stress can increase only if the neutral axis rises and the compressive force is diminished, or vice versa. Obviously a theory which will predict the shear-compression strength of the beams of this investigation must be such that is allows both tension and compression forces to increase.

In section 10(a) it was pointed out that the steel strain measurements indicated that the stress in the reinforcement after diagonal tension cracking was almost constant over a considerable portion of the span. Thus the beams acted like tied arches and there is no reason why an increase in the tension force at failure should necessarily involve a reduction of the compressive force.

Since the axial load was kept constant at all times, the ratio of axial load to shear varied. The values at ultimate for the shear-compression failures ranged from 0.62 for beam B-11 to 2.83 for beam B-3. The value of the ratio does not appear to have much significance, however, since the amount of increase in moment due to axial load is roughly the same in spite of the variation in the axial load to shear ratio.

14. Flexural Failures

The comparison between the computed and measured ultimate moments for beams A-5 and B-5 which failed in flexure is shown in Table 7. The equations from which the ultimate flexural moments were computed are given in the table. The equation for ultimate moment is equally valid for beams with and without axial load since for beams without axial load $N = 0$, and the equation reduces to the usual one for beams at flexural ultimate. The value of e_u, $k_1 k_3$, and k_2 were assumed as indicated in the table, and represent values which have been found in other studies to give reliable results. Since the value of the deflection at ultimate had to be known for beam B-5, the measured value of 1.0 in. was used.

The measured values of the ultimate moment were in good agreement with the computed values. Since the computed values were 6 per cent higher in both cases, the equations used appear to be satisfactory in so far as the effect of axial load on the ultimate flexural moment is concerned. It also appears that the inclined cracks which developed near failure did not appreciably affect the load carrying capacity.

15. Comparison With Previous Test Results

A previous report (Ref. 4) presented the results of tests on six beams tested under two point loading with no axial load. The cross-sectional properties and the shear span of those beams were the same as beams A-11 through A-15 reported here except that in the previous report one longer beam which failed in flexure was tested. The purpose of this section is to compare the results of the previous tests with those reported here.

The beams for which a comparison can be made are listed in Table 8. Beams L-1 through L-6 are from the previous report and beams A-11 through A-15 are from the present investigation for which the comparison is to be made.

In order to present the data in a more consistent manner, it is necessary to adjust the measured values of moment for the variation in concrete strength. This is done in Table 8 by multiplying the measured values of ultimate moment by the factor K. The factor K adjusts the measured values to values corresponding to a concrete strength of 4000 psi, and is the ratio of the calculated value of shear moment from equation (4) for a beam with 4000 psi, to the calculated shear moment for the beam with f'_c equal to the respective values for each beam. The adjusted values of ultimate moment appear under the heading KM_u in Table 8, and it is these values which are to be compared.

In Fig. 20 the beams are compared graphically. The adjusted values of moment are plotted against the shear span-depth ratio a/d. Also included on the figure are the computed value of M_s, the moment corresponding to first cracking according to equation (2), and the ultimate flexural moment.

The beams failing in diagonal tension have already been compared in the section on diagonal tension cracking, and were found to be in good agreement with the results of these tests. This is illustrated again in Fig. 20.

The shortest beams, with a/d equal to 2, can be seen to be in good agreement with each other and with M_s. Beam A-12 of this investigation with a/d equal to 3 failed in diagonal tension at a moment 14 per cent less than M_s, while the two corresponding beams of the previous report failed in shear-compression at ultimate moments somewhat greater than M_s.

The beams tested previously had an external stirrup placed just inside the supports, while for the beams of this report the stirrup was placed just outside the supports. It is believed that this difference may account for the greater loads carried by the short beams of the previous investigation. It may also explain why the transition to diagonal tension failure was not at the same a/d ratio for the two investigations.

Only one flexural failure is presented in Table 8 and Fig. 20. It was the longest beam of those in Ref. 4. The value of ultimate moment was not corrected for f'_c in the table, since the flexural strength does not depend on concrete strength to such a large extent. The measured ultimate moment was 2 per cent less than the computed value.

V. SUMMARY

The object of this investigation was to study the effect of axial load on the shear strength of reinforced concrete beams.

Twenty beams were tested, ten without axial load and ten with an axial load of 20 kips. Except for small variations in concrete strength, the beams tested with axial load were identical to those tested without axial load. The other variables were steel percentage and span length. Only two steel percentages were used, 0.0100 and 0.0333. The span length was varied from 52 to 132 in., in 20-in. increments so that the shear span-depth ratio varied from 2 to 6. Four beams were tested on each span length, one at each steel percentage with and without axial load.

The beams were loaded in several increments up to failure. The load was applied at midspan through an integrally cast column stub. For those beams tested under axial load, the axial load was applied at the beginning of the test and was kept constant at 20 kips throughout the test. Loads and deflections were measured in all of the beams and strains in the tensile steel were measured in some.

Three modes of failure were observed, shear-compression, diagonal tension and flexure. Eight beams failed in shear-compression, ten in diagonal tension and two in flexure. The shear-compression failures occurred in the shortest beams, the diagonal tension failures occurred in the medium long beams and the flexural failures occurred in the two longest beams with the small steel percentage.

1. Diagonal Tension Cracking

The diagonal tension cracking load was observed in all beams except in those failing in flexure. For the beams which failed in diagonal tension, the diagonal tension cracking load was also the ultimate load; however, for those which failed in shear-compression, the ultimate load was greater than the diagonal tension cracking load.

The following empirical equation was developed for the nominal unit shearing stress at diagonal tension cracking:

$$v_c = \frac{120\ p + 4.3}{23 + 2\ a/d} \cdot \frac{f'_c}{1 + \dfrac{0.85\ f'_c}{1000}} + \frac{N}{\frac{7}{8}\ bd}\,(0.270 - 3.4\ p - 0.034\ a/d) \qquad (3)$$

Equation (3) is equally good for beams with and without axial load. The correlation of the tests results with equation (3) was within 5 per cent except for three beams. Two of these were the shortest beams tested with axial load, for which error was 15 and 16 per cent. It is felt that the behavior of very short beams is somewhat different than for longer ones. Nevertheless, the equation gives values lower than those measured for the short beams and thus is on the safe side. The effects of the concrete strength f'_c and the steel percentage p for beams without axial load were taken from other studies and were found to be satisfactory in so far as this investigation is concerned. The effect of axial load was to increase the diagonal tension strength. The amount of the increase was found to be less for the beams with a high steel percentage and for the longer beams. For the longest beams with the high steel percentage, the axial load did not increase the diagonal tension cracking strength.

Equation (3) applies only to beams with an a/d ratio equal to or greater than 2. Other studies have indicated that the diagonal tension cracking strength increases considerably for very short beams; consequently, equation (3) would require modification to apply to such beams.

Since only one value of axial load and only two steel percentages were used, and since the concrete strength was kept constant, the second term of equation (3) which represents the additional strength due to axial load cannot be considered general.

Equation (3) was developed for beams loaded at midspan through a stub but was found to apply equally well to beams loaded with two symmetrically placed concentrated loads. Thus the stub does not appear to affect the diagonal tension cracking strength.

2. Shear-Compression Failures

Two of the beams tested without axial load, and six of those tested with axial load failed in shear-compression. The moment at ultimate load for the beams without axial load was in fairly good agreement with the equation:

$$M_s = bd^2 f'_c k \left(0.57 - \frac{4.5\, f'_c}{10^5}\right) \qquad (4)$$

The axial load increased the shear-compression strength more than the diagonal tension strength with the result that the mode of failure was changed for some of the beams. For four of the beams which failed in shear-compression with axial load, the corresponding beams without axial load failed in diagonal tension. The increase in strength was larger for the shorter beams than for the longer ones. The increase in moment was in all cases more than could be accounted for by the axial load directly. Both the total compression force in the concrete and the total tension force in the steel were larger for the beams tested with axial load.

The increase in strength due to axial load did not seem to be a function of the ratio of axial load to shear.

Strain measurements in the steel indicated that there was a major redistribution of stress after diagonal tension cracking, which resulted in the stress in the steel being constant over a considerable portion of the span. It is suggested that, since beams undergo considerable damage at diagonal tension cracking, and since the shear-compression strength is not as predictable as the diagonal tension cracking strength, design considerations should perhaps be based on diagonal tension cracking.

3. Flexural Failures

The measured values of the ultimate moment for the flexural failures were found to be in good agreement with the computed values. The effect of the axial load was to increase the load-carrying capacity.

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TABLE 1

PROPERTIES OF BEAMS

Beam	L in.	a/d	f'_c (psi) Batch 1	f'_c (psi) Batch 2	f_y* ksi	Stirrups
For all beams: b = 6 in., d = 10 in., h = 12 in.						
(a) Beams reinforced with three No. 4 bars, p = 0.0100						
A-1	52	2	4230	4070	66.5	yes
A-2	72	3	4450	4570	68.0	no
A-3	92	4	3220	2820	65.6	yes
A-4	112	5	3490	3890	66.6	yes
A-5	132	6	4270	4450	67.0	yes
B-1	52	2	4350	3780	66.5	yes
B-2	72	3	3900	4140	66.5	yes
B-3	92	4	4080	3820	67.1	yes
B-4	112	5	3790	4100	66.5	yes
B-5	132	6	4190	4120	67.0	yes
(b) Beams reinforced with two No. 9 bars, p = 0.0333						
A-11	52	2	3830	4100	49.5	yes
A-12	72	3	4180	3870	45.5	yes
A-13	92	4	2960	3210	57.0	yes
A-14	112	5	3780	3990	52.8	yes
A-15	132	6	4070	3630	48.1	yes
B-11	52	2	3940	3660	48.1	yes
B-12	72	3	3800	3930	56.8	yes
B-13	92	4	4150	4050	51.4	yes
B-14	112	5	3880	4250	52.6	yes
B-15	132	6	4200	4110	47.3	yes

Note: A - series without axial load
B - series with axial load of 20 kips
* - the values of f_y are the average value

TABLE 2(a)

PROPERTIES OF CONCRETE MIXES
BEAMS REINFORCED WITH THREE NO. 4 BARS

Beam	Batch	Cement:Sand:Gravel by weight	Cement/Water by weight	Slump in.	Compressive Strength, f'_c psi	Modulus of Rupture, f_r psi	Age at Test Days
A-1	1	1.00:3.39:5.05	1.29	1	4230	383	32
	2	1.00:3.44:5.09	1.23	2 1/2	4070	417	32
A-2	1	1.00:3.49:5.15	1.35	2	4450	500	35
	2	1.00:3.48:5.10	1.45	3	4570	550	35
A-3	1	1.00:3.49:5.13	1.21	1	3220	467	35
	2	1.00:3.49:5.13	1.22	5	2820	467	35
A-4	1	1.00:3.48:5.13	1.05	5	3490	575	28
	2	1.00:3.50:5.60	1.22	3	3890	642	28
A-5	1	1.00:3.46:5.07	1.17	2	4270	383	32
	2	1.00:3.49:5.11	1.22	1	4450	383	32
B-1	1	1.00:3.46:5.13	1.38	2	4350	525	33
	2	1.00:3.51:5.16	1.37	4	3780	458	33
B-2	1	1.00:3.51:5.12	1.30	2	3900	483	29
	2	1.00:3.51:5.07	1.33	2 1/2	4140	475	29
B-3	1	1.00:3.51:5.15	1.30	2	4080	375	30
	2	1.00:3.49:5.17	1.28	3	3820	475	30
B-4	1	1.00:3.44:5.11	1.24	6	3790	408	27
	2	1.00:3.46:5.13	1.43	2	4100	458	27
B-5	1	1.00:3.44:5.13	1.29	2	4190	542	36
	2	1.00:3.47:5.20	1.29	2	4120	542	36

TABLE 2(b)

PROPERTIES OF CONCRETE MIXES
BEAMS REINFORCED WITH TWO NO. 9 BARS

Beam	Batch	Cement:Sand:Gravel by weight	Cement/Water by weight	Slump in.	Compressive Strength, f'_c psi	Modulus of Rupture, f_r psi	Age at Test Days
A-11	1	1.00:3.47:5.19	1.22	2	3830	417	35
	2	1.00:3.51:5.23	1.33	1	4100	417	
A-12	1	1.00:3.47:5.19	1.34	3	4180	467	29
	2	1.00:3.47:5.15	1.38	4	3870	483	
A-13	1	1.00:3.57:5.23	1.02	2 1/2	2960	350	28
	2	1.00:3.51:5.24	1.31	2 1/2	3210	425	
A-14	1	1.00:3.41:5.16	1.28	3	3780	433	26
	2	1.00:3.42:5.17	1.42	2	3990	458	
A-15	1	1.00:3.39:5.07	1.17	5	4070	508	42
	2	1.00:3.39:5.08	1.13	3	3630	450	
B-11	1	1.00:3.47:5.07	1.09	2	3940	358	35
	2	1.00:3.51:5.08	1.25	3	3660	450	
B-12	1	1.00:3.52:5.09	1.21	3	3800	442	35
	2	1.00:3.50:5.05	1.27	6	3930	458	
B-13	1	1.00:3.43:5.11	1.16	3 1/2	4150	425	31
	2	1.00:3.39:5.06	1.19	2 1/2	4050	408	
B-14	1	1.00:3.40:5.10	1.12	6	3880	408	29
	2	1.00:3.21:4.96	1.33	6	4250	433	
B-15	1	1.00:3.52:5.14	1.34	2	4200	400	28
	2	1.00:3.48:5.09	1.35	2	4110	408	

TABLE 3

DISTANCES TO IMPORTANT POINTS ON CRACK CAUSING FAILURE

Beam No.	a in.	x_x in.	$k_u d$ in.
	(a) Beams Reinforced With Three No. 4 Bars		
	No Axial Load		
A-1	20	6.75	0.75
A-2	30	10.0	1.1
A-3	40	16.25	0.8
A-4	50	29.5	0
A-5	60	Flex.	1.5
	20-kip Axial Load		
B-1	20	6.0	1.7
B-2	30	15.75	1.3
B-3	40	22.0	2.2
B-4	50	28.0	1.8
B-5	60	Flex.	3.0
	(b) Beams Reinforced With Two No. 9 Bars		
	No Axial Load		
A-11	20	4.0	1.1
A-12	30	10.5	1.3
A-13	40	12.2	1.2
A-14	50	19.0	1.5
A-15	60	39.0	3.2
	20-kip Axial Load		
B-11	20	9.75	2.7
B-12	30	13.75	3.4
B-13	40	23	2.5
B-14	50	28	3.0
B-15	60	38.0	3.6

TABLE 4

TEST RESULTS

Beam No.	$\frac{a}{d}$	Axial Load kips	Mode* of Failure	Cracking Load P_c kips	Ultimate Load P_u kips	M_u $\frac{P_u a}{2}$ in-kips	Maximum steel Stress ksi	f'_c (Batch 2) psi
				(a) Beams reinforced with three No. 4 bars				
				No Axial Load				
A-1	2	0	S	20.6	33.0	330		4070
A-2	3	0	DT	18.8	18.8	282		4570
A-3	4	0	DT	15.4	15.4	308		2820
A-4	5	0	DT	15.8	15.8	395		3890
A-5	6	0	F	----	14.7	441		4450
				Axial Load = 20 kips				
B-1	2	20	S	29.8	51.4	514		3780
B-2	3	20	S	23.4	29.5	442		4140
B-3	4	20	S	20.6	21.8	436		3820
B-4	5	20	DT	18.9	18.9	472		4100
B-5	6	20	F	----	15.1	453		4120
				(b) Beams reinforced with two No. 9 bars				
				No Axial Load				
A-11	2	0	S	28.3	46.5	465	24.4	4100
A-12	3	0	DT	26.5	26.5	397	20.7	3870
A-13	4	0	DT	21.1	21.1	422	26.6	3210
A-14	5	0	DT	24.6	24.6	615	30.2	3990
A-15	6	0	DT	22.2	22.2	666	41.6	3630
				Axial Load = 20 kips				
B-11	2	20	S	37.8	64.5	645	31.4	3660
B-12	3	20	S	29.6	33.7	505	19.7	3930
B-13	4	20	S	26.5	27.8	556	30.5	4050
B-14	5	20	DT	23.8	23.8	595	26.0	4250
B-15	6	20	DT	21.1	21.1	630	39.5	4110

* S - Shear-Compression Failure
DT - Diagonal Tension Failure
F - Flexural Failure

TABLE 5

COMPARISON OF MEASURED AND COMPUTED VALUES
OF NOMINAL UNIT SHEARING STRESS AT DIAGONAL TENSION CRACKING

Measured values include live load only

Beam No.	Computed* v_c psi	Measured v_c psi	Ratio $\frac{\text{Measured}}{\text{Computed}}$
(a) Beams with no Axial Load			
A-1	186	196	1.05
A-2	177	179	1.01
A-3	147	147	1.00
A-4	151	151	1.00
A-11	281	270	0.96
A-12	258	252	0.98
A-13	231	201	0.87
A-14	229	234	1.02
A-15	211	211	1.00
		Average	0.988
		Average Error	0.03
(b) Beams with 20-kip Axial Load			
B-1	247	284	1.15
B-2	225	223	0.99
B-3	198	196	0.99
B-4	177	180	1.02
B-11	308	360	1.16
B-12	280	282	1.01
B-13	252	252	1.00
B-14	227	227	1.00
B-15	199	200	1.00
		Average	1.035
		Average Error	0.04

* Computed values from equation (3) which reduces to equation (2) for beams with no axial load

TABLE 6

COMPUTED STEEL STRESSES FOR SHEAR-COMPRESSION FAILURES

Beam No.	$V_u a$ in-kips	$k_u d$ * in.	y in.	$N(\frac{h}{2} - y - \frac{k_u d}{2})$ ** in-kips	$A_s f_s d(1-\frac{k_u}{2})$ in-kips	f_s comp. ksi	f_s meas. ksi
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
A-1	330	0.75		0	330	57.0	
B-1	514	1.7	0.25	98	416	75.5	
A-2	282	1.1		0	282	49.7	
B-2	442	1.3	0.37	100	342	60.8	
A-3	308	0.8		0	308	53.4	
B-3	436	2.2	0.33	91	345	64.5	
A-11	465	1.1		0	465	24.5	24.4
B-11	645	2.7	0.25	88	557	32.2	31.4
A-12	397	1.3		0	397	21.2	20.7
B-12	505	3.4	0.22	82	423	25.4	19.7
A-13	422	1.2		0	422	22.5	26.6
B-13	556	2.5	0.56	84	472	27.0	30.5

* $k_u d$ = measured value from Table 3

** $k_2 = 0.50$

TABLE 7

COMPARISON OF MEASURED AND COMPUTED VALUES
OF ULTIMATE MOMENT FOR FLEXURAL FAILURES

Beam No.	y in.	e_s in./in.	f_s* ksi	C kips	M_f in.-kips Comp.	M_u in.-kips Meas.	Ratio Measured/Computed
A-5		.0143	86.0	51.7	469	441	0.94
B-5	1.0	.0094	77.3	66.4	481	453	0.94

* Since the steel was in the strain-hardening range, the stress-strain curve was used to determine the steel stress.

$$M_f = C\,d\,(1 - k_2 k_u) - N\,(d - \frac{h}{2} + y)$$

where:

$$k_1 k_3 = \frac{3000 + 0.5\,f'_c}{1500 + f'_c}$$

$$A_s = 0.60 \text{ in.}^2$$

$$e_u = 0.004 \text{ in./in.}$$

$$b = 6 \text{ in.}$$

$$d = 10 \text{ in.}$$

$$k_2 = 0.42$$

$$h = 12 \text{ in.}$$

TABLE 8

COMPARISON WITH TEST RESULTS FOR BEAMS OF REFERENCE (4)

Beam No.	a/d	f'_c psi	M_u in-kips	K *	KM_u in-kips	Mode of Failure
L-1	2	3050	444	1.152	511	S
L-2	3	3120	459	1.124	516	S
L-2a	3	5320	540	0.915	494	S
L-3	4	4060	480	0.985	473	DT
L-4	5	3740	575	1.029	592	DT
L-5	6	4050	690	0.987	681	DT
L-6	7	4440	735	-	-	F
A-11	2	4100	465	0.987	459	S
A-12	3	3870	397	1.018	404	DT
A-13	4	3210	422	1.124	474	DT
A-14	5	3990	615	1.002	616	DT
A-15	6	3630	666	1.056	703	DT

$$* \quad K = \frac{M_s\ (f'_c = 4000 \text{ psi})}{M_s\ (\text{Actual } f'_c)}$$

Note: Beam L-1 through L-6 from Table 5, Ref. 4.

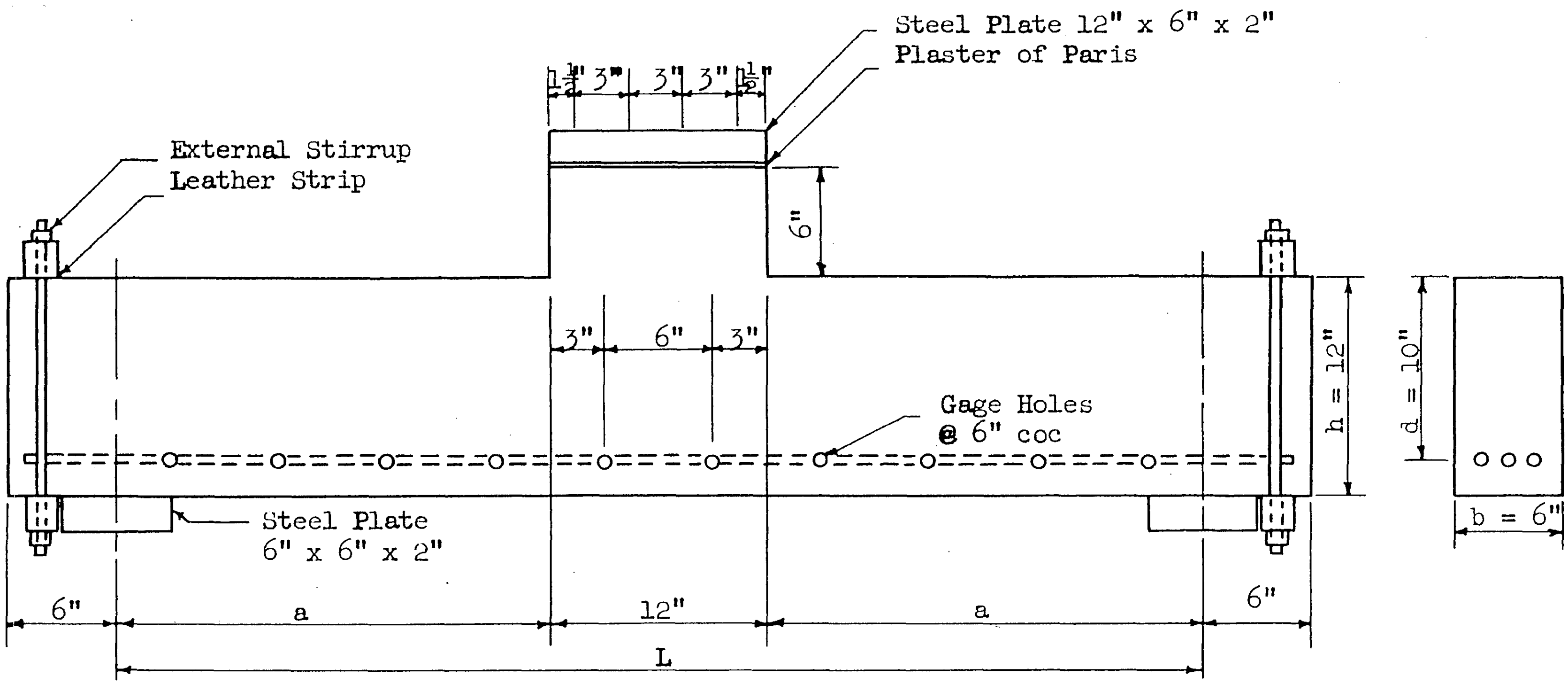


FIG. 1 DIMENSIONS OF TEST BEAMS

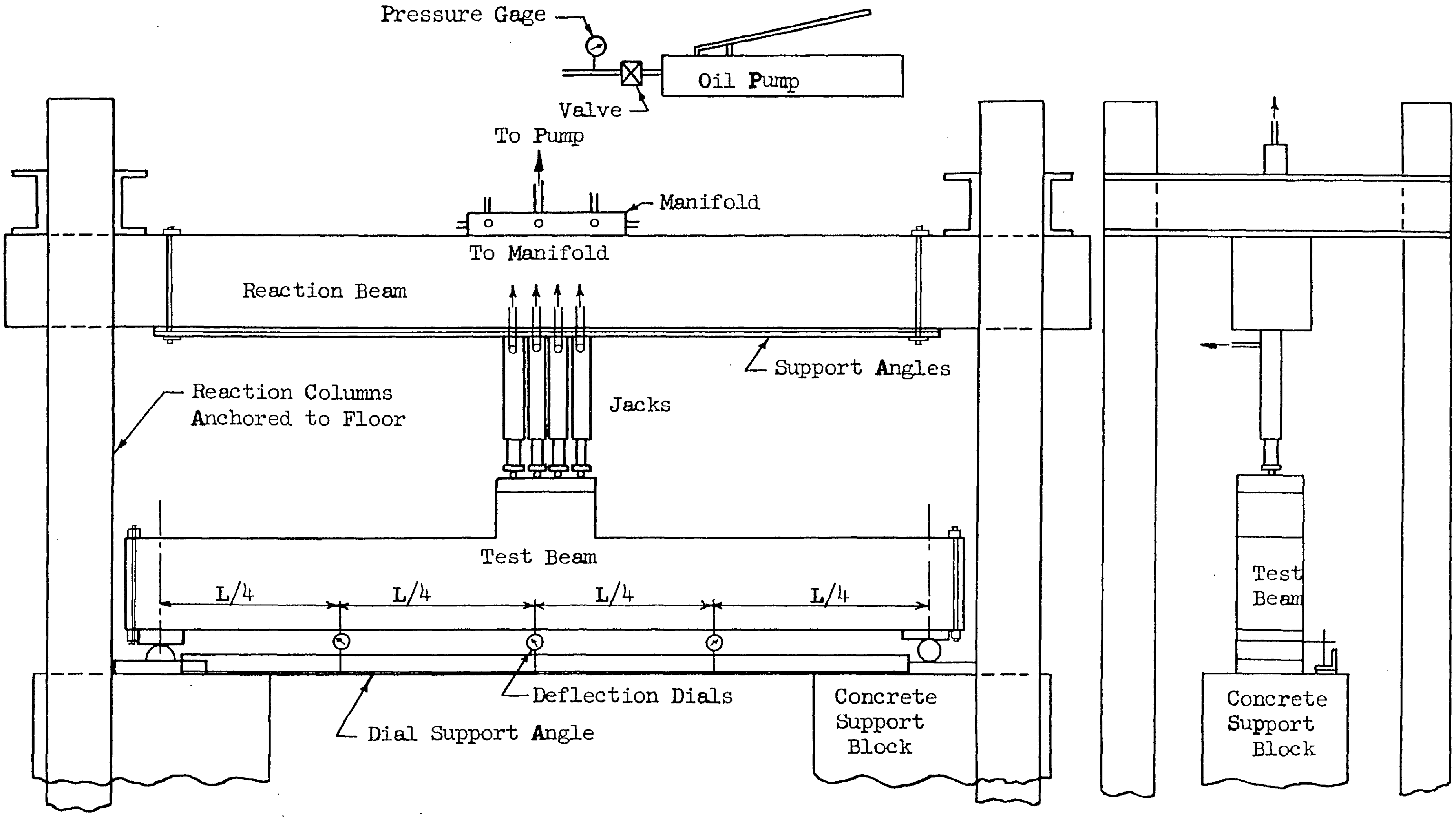


FIG. 2 TYPICAL TEST SETUP

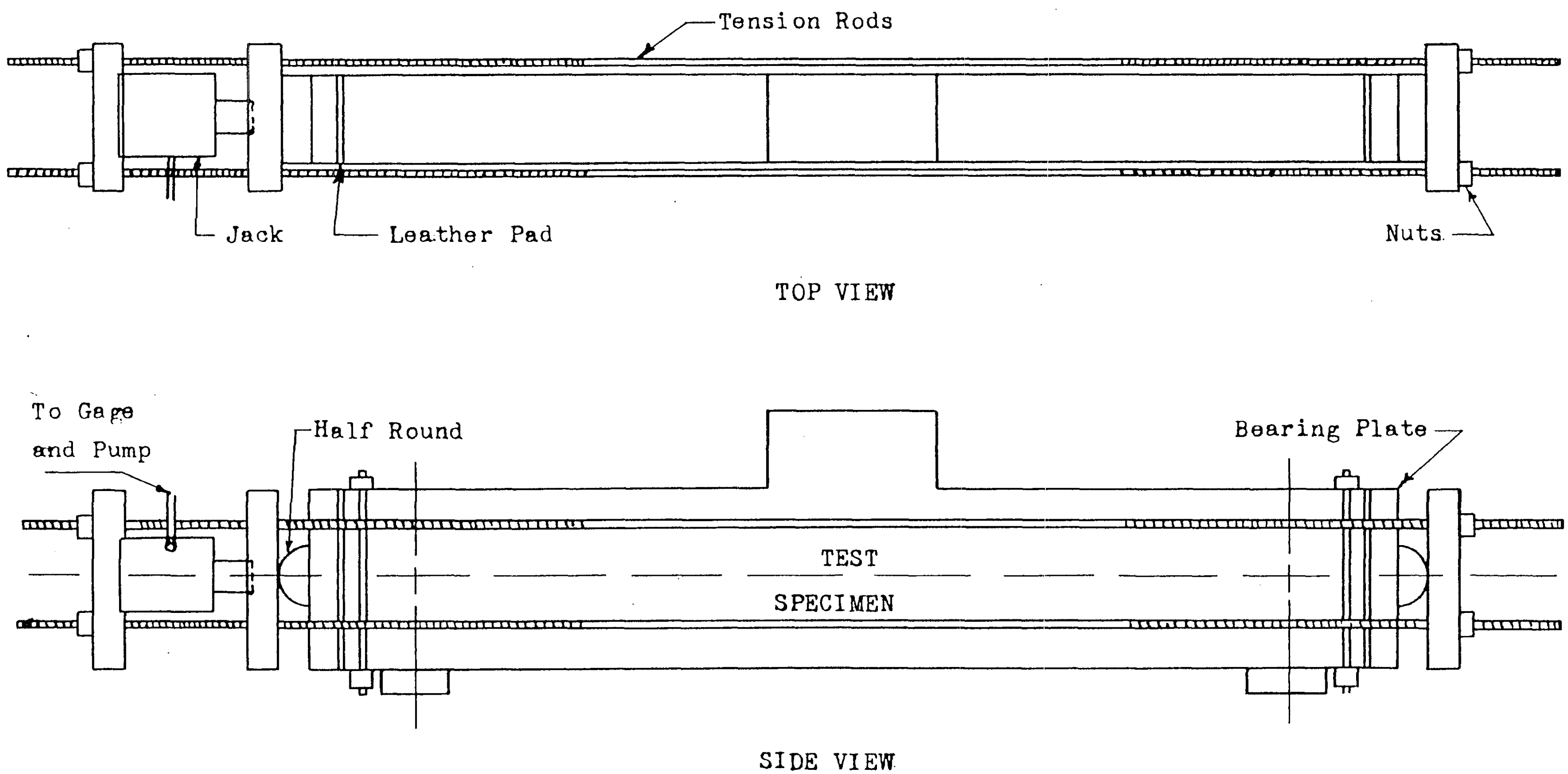


FIG. 3 AXIAL LOADING EQUIPMENT

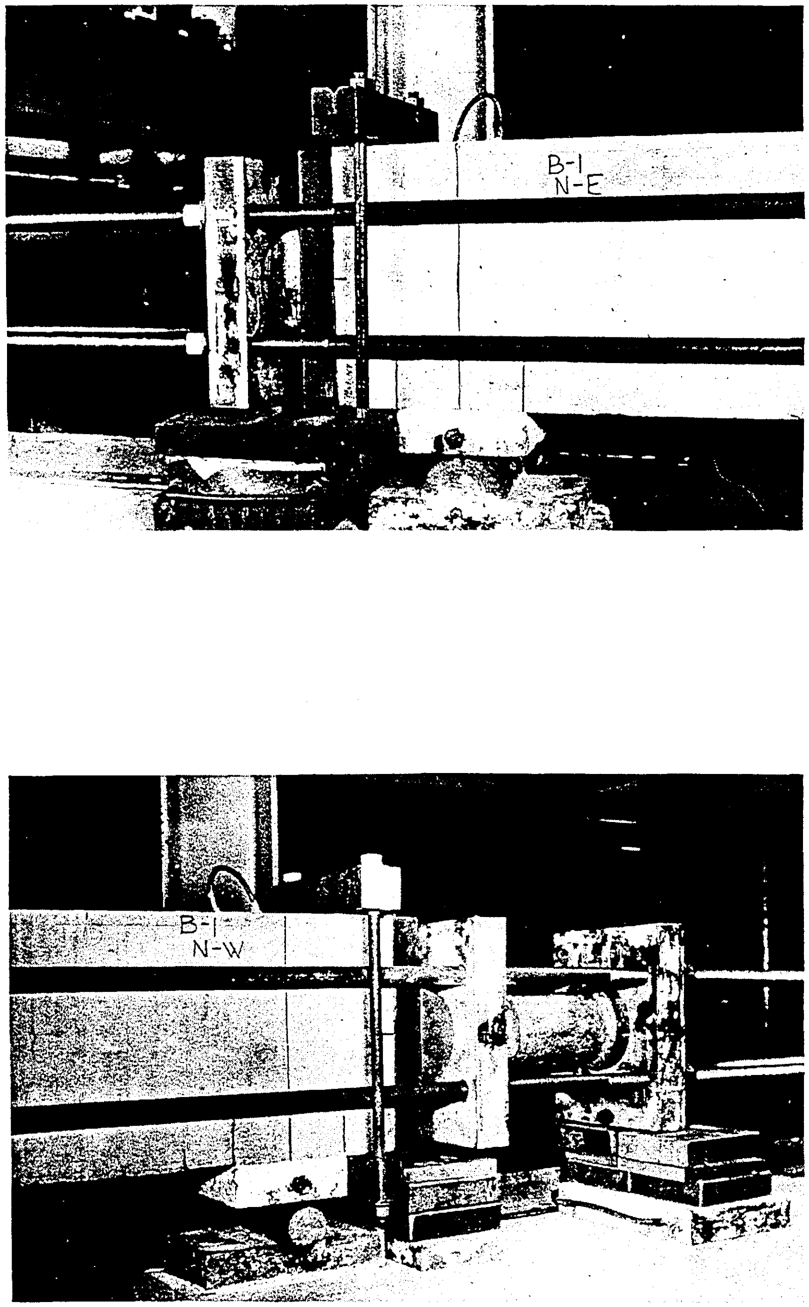


FIG. 4 PHOTOGRAPHS OF AXIAL LOADING EQUIPMENT

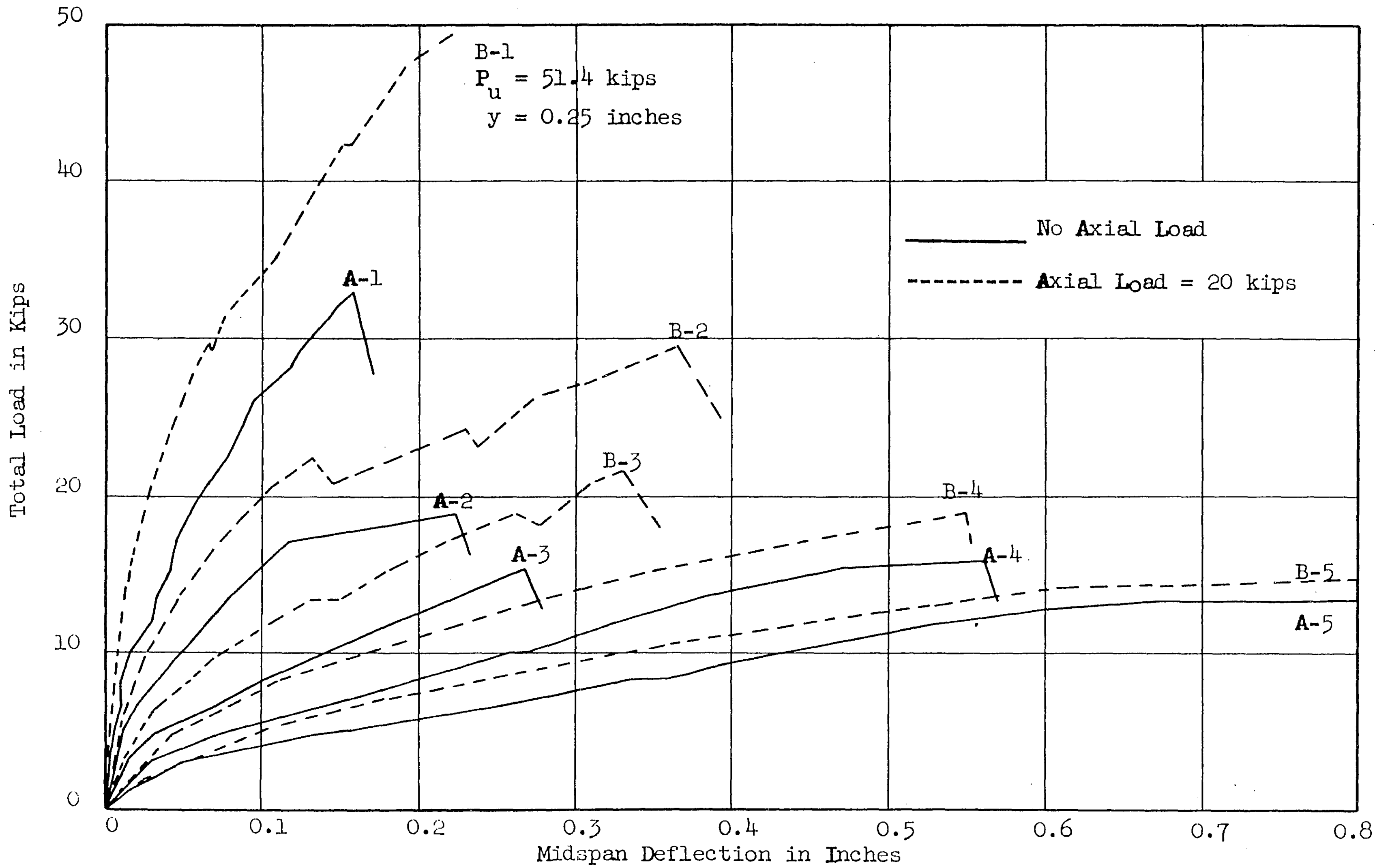


FIG. 5 LOAD-DEFLECTION CURVES FOR BEAMS WITH THREE NO. 4 BARS

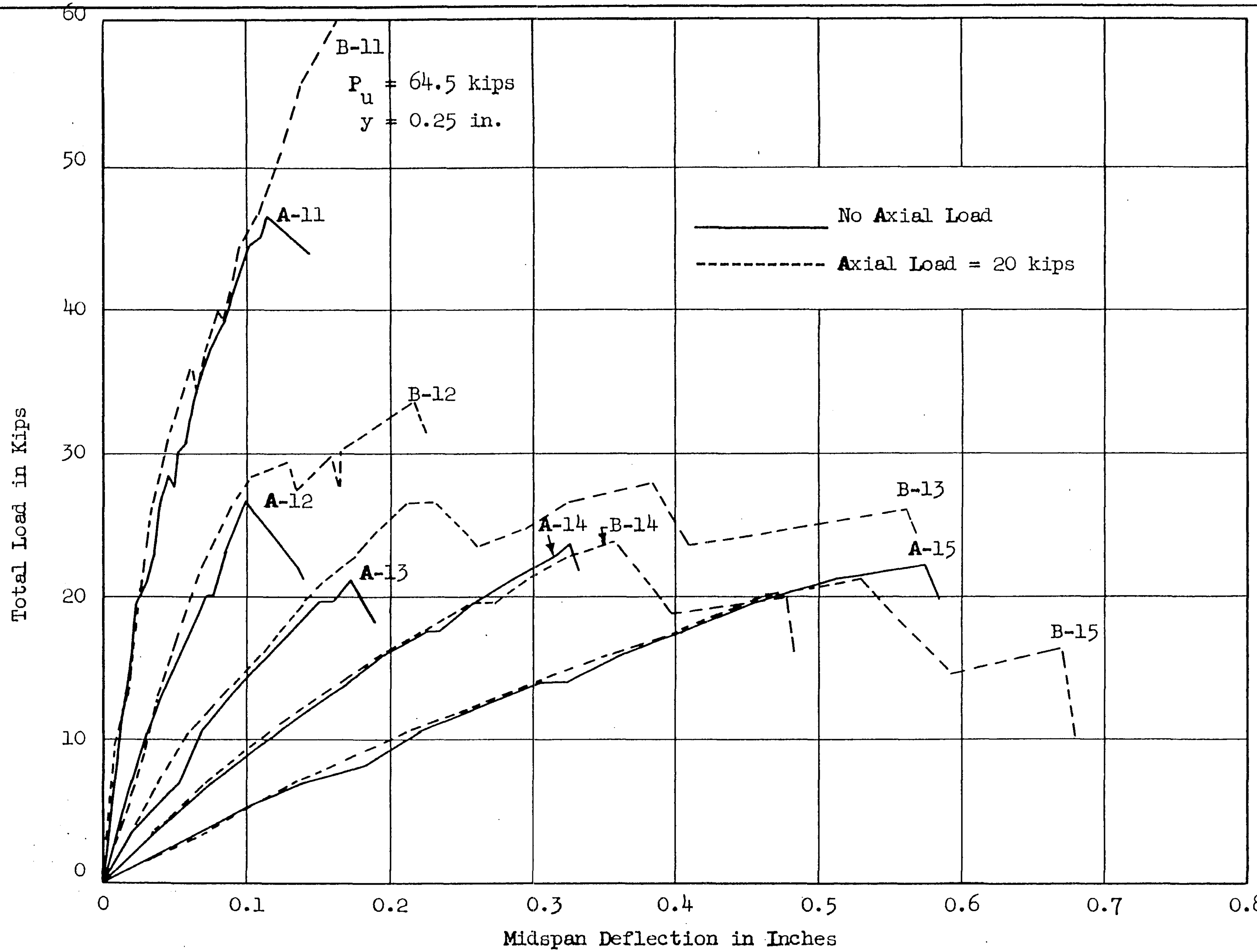


FIG. 6 LOAD-DEFLECTION CURVES FOR BEAMS WITH TWO NO. 9 BARS

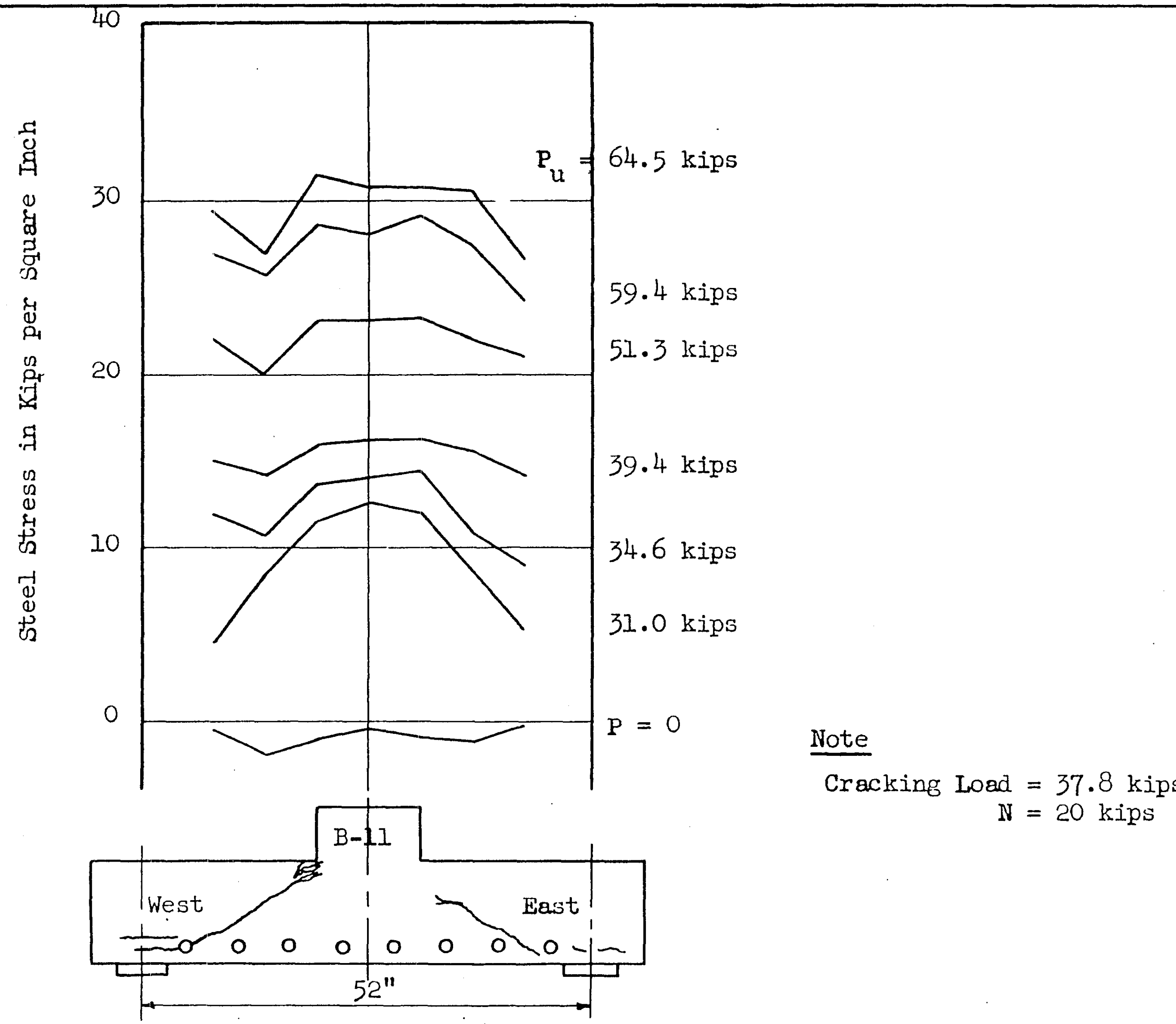
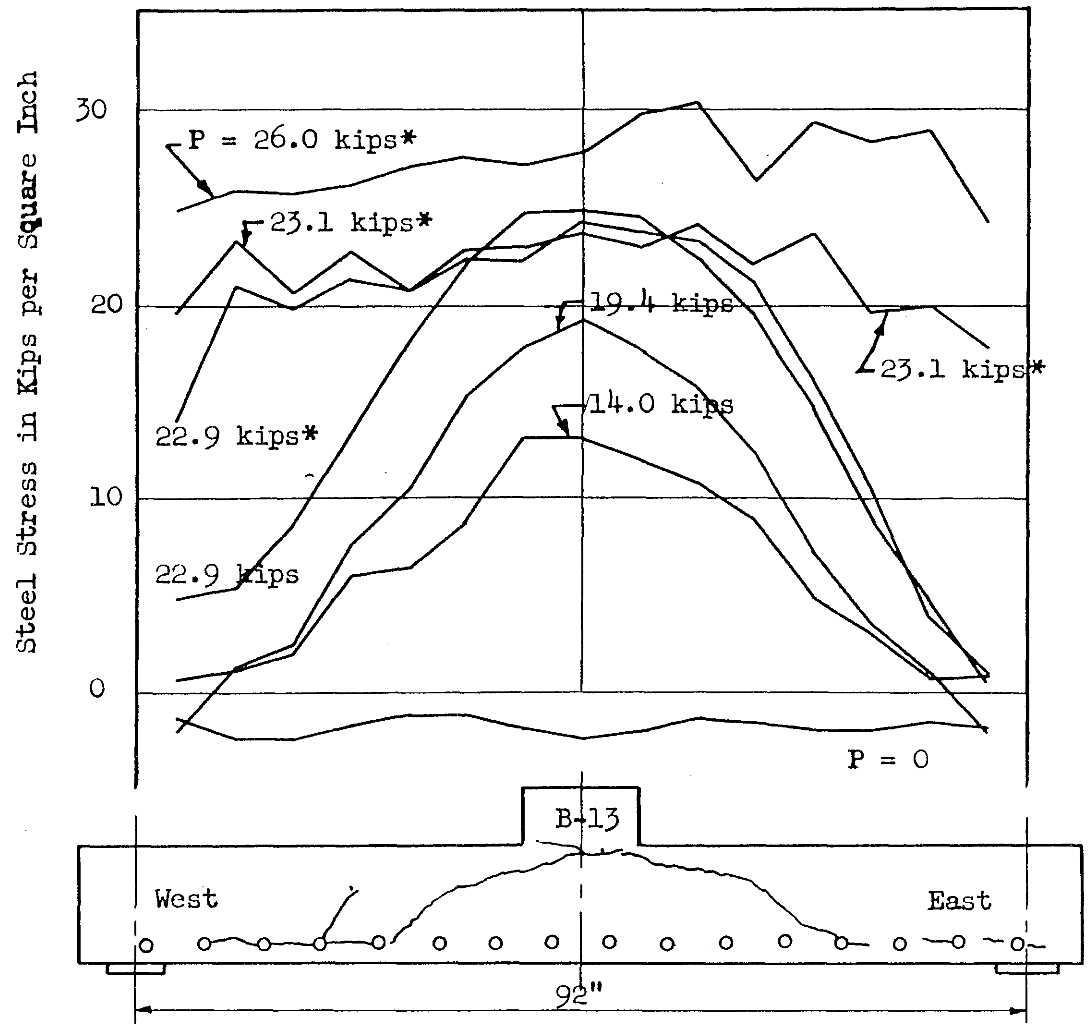


Fig. 7 STEEL STRESS DISTRIBUTION FOR SHEAR COMPRESSION FAILURE. BEAM B-11



NOTE

* Measured after end cracked and load dropped

East end cracked at 27.8 kips

West end cracked at 26.5 kips

N = 20 kips

FIG. 8 STEEL STRESS DISTRIBUTION FOR SHEAR COMPRESSION FAILURE. BEAM B-13

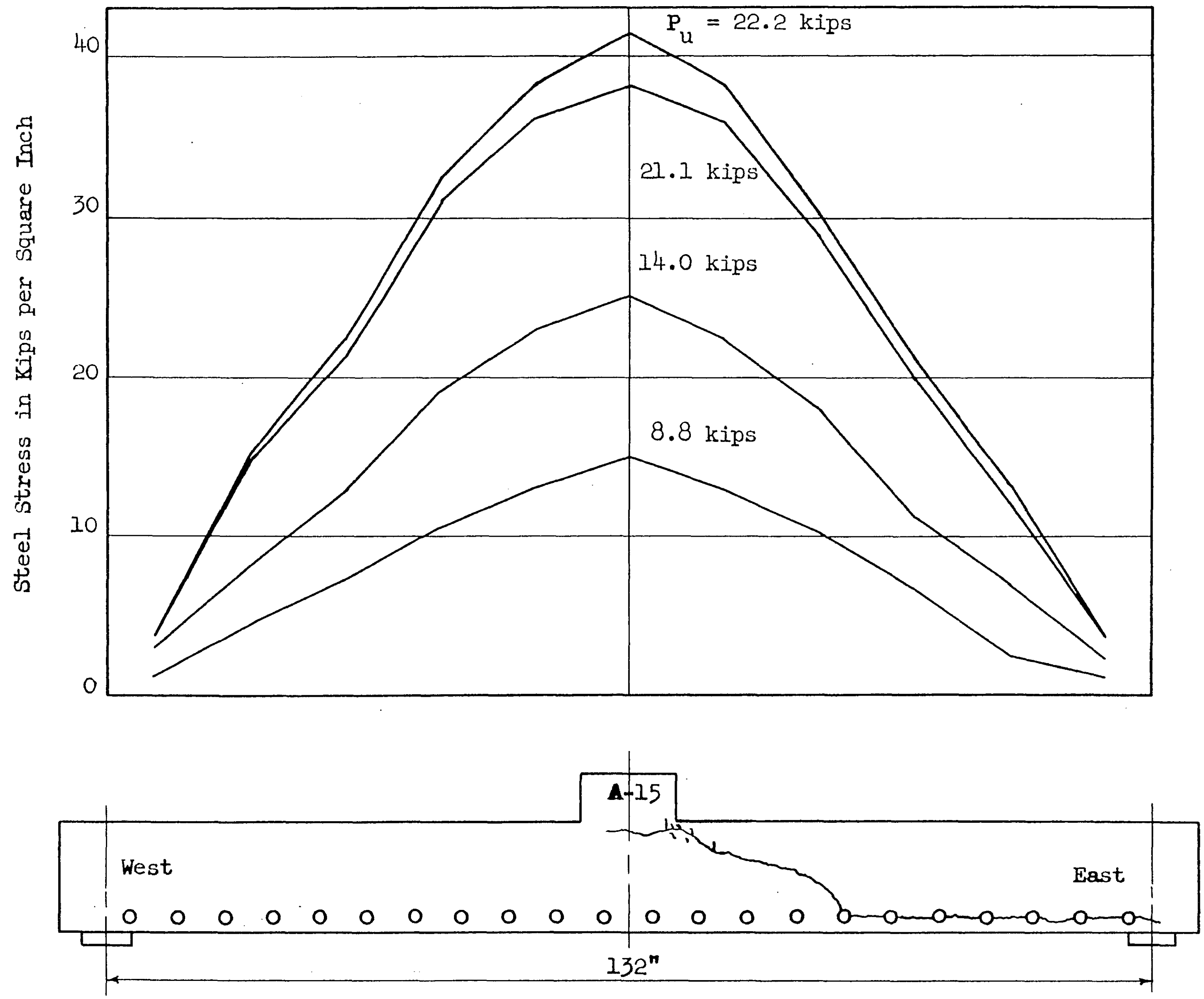
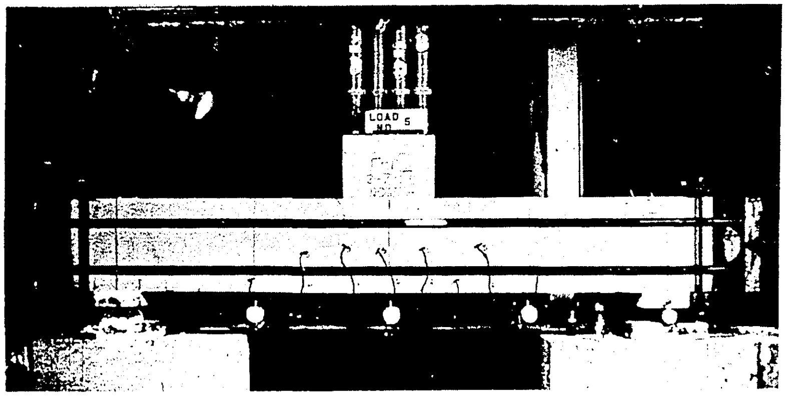
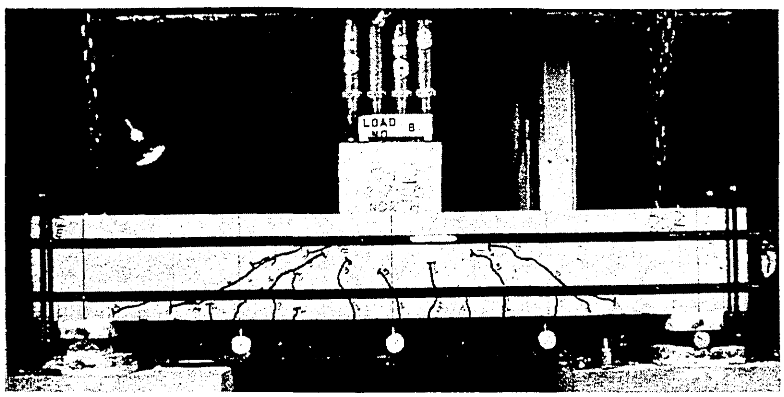


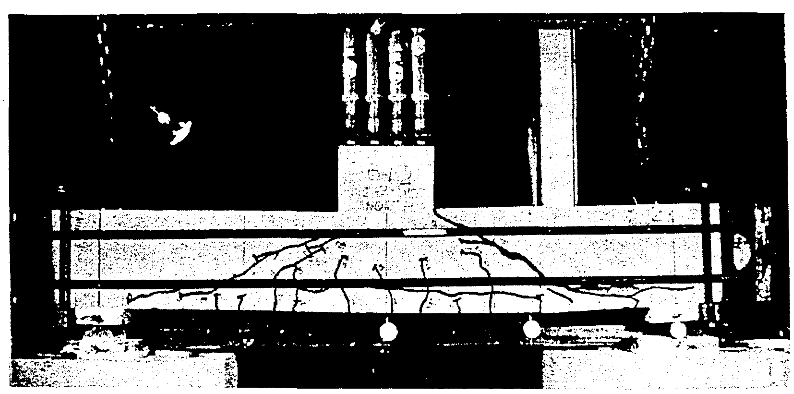
FIG. 9 STEEL STRESS DISTRIBUTION FOR DIAGONAL TENSION FAILURE. BEAM A-15



(a) P = 22.0 kips

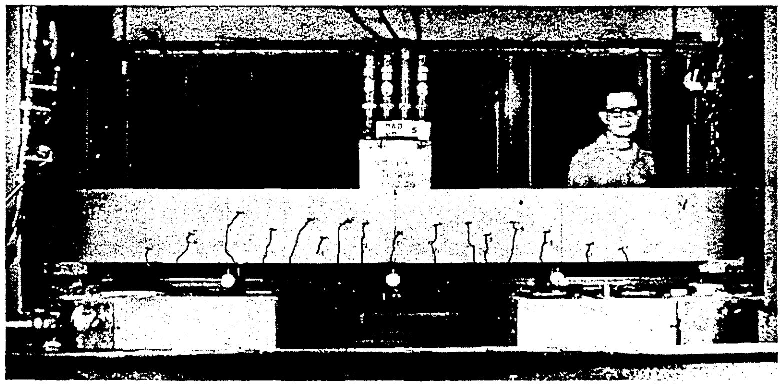


(b) P = 29.6 kips

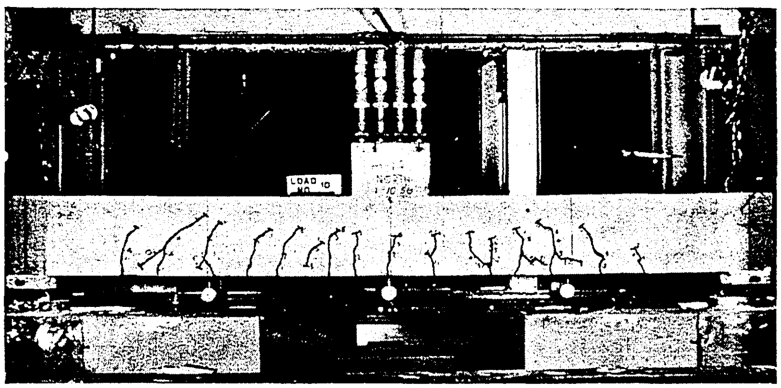


(c) P_u = 33.7 kips (after failure)

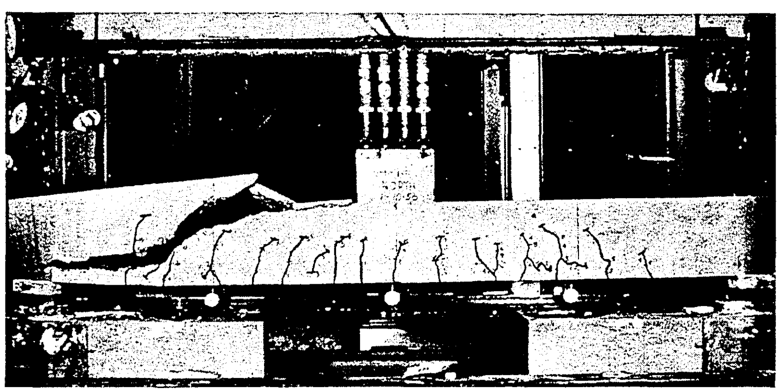
FIG. 10 CRACK DEVELOPMENT FOR SHEAR-COMPRESSION FAILURE. BEAM B-12



(a) $P = 14.0$ kips

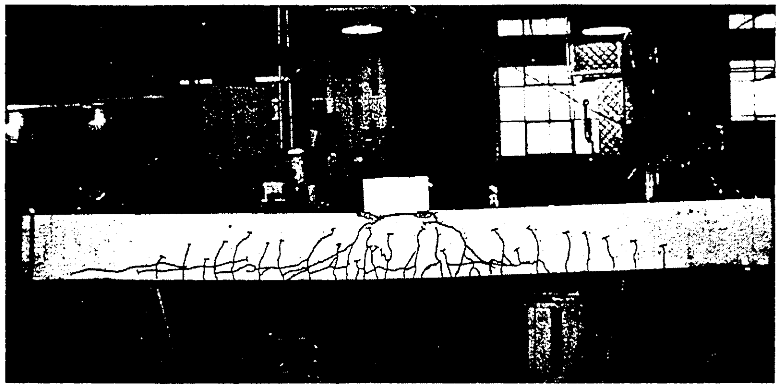


(b) $P = 22.9$ kips

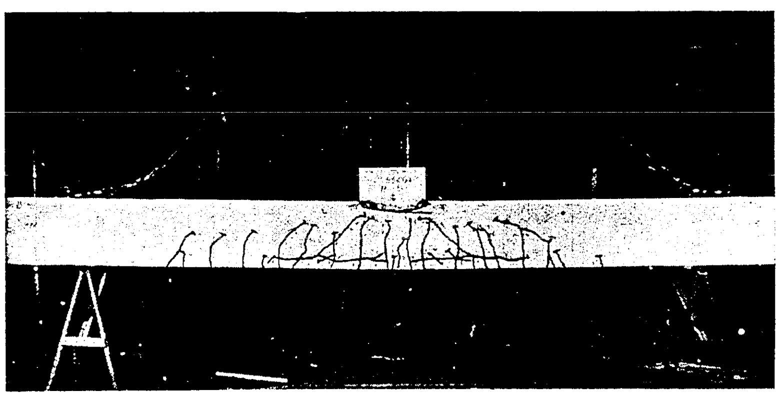


(c) $P_u = 24.6$ kips (after failure)

FIG. 11 CRACK DEVELOPMENT FOR DIAGONAL TENSION FAILURE. BEAM A-14

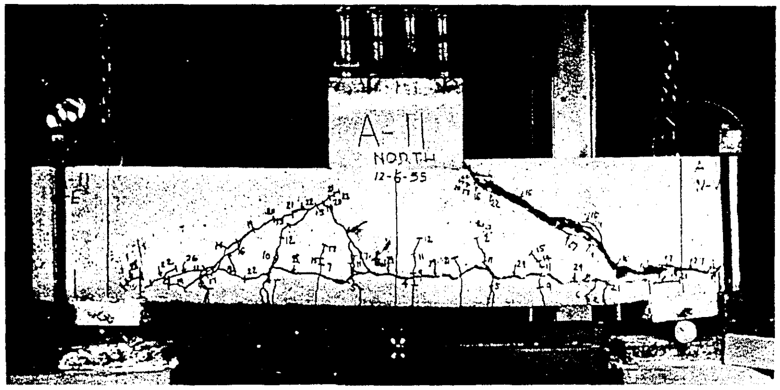


(a) Beam A-5 without axial load. P_u = 14.7 kips.

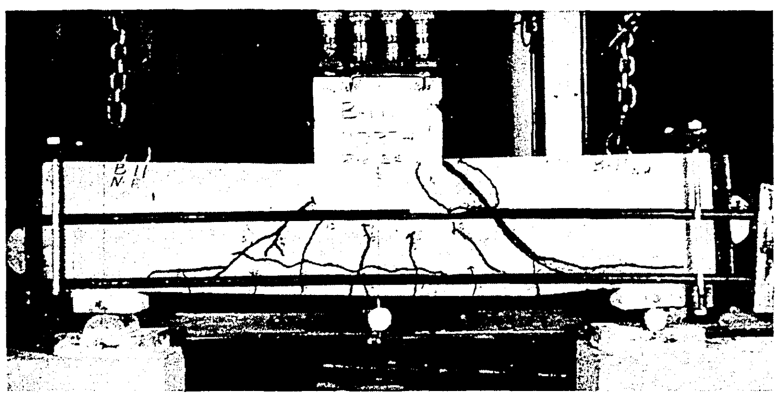


(b) Beam B-5 with axial load. P_u = 15.1 kips.

FIG. 12 EFFECT OF AXIAL LOAD ON CRACKING PATTERN FOR FLEXURAL FAILURE

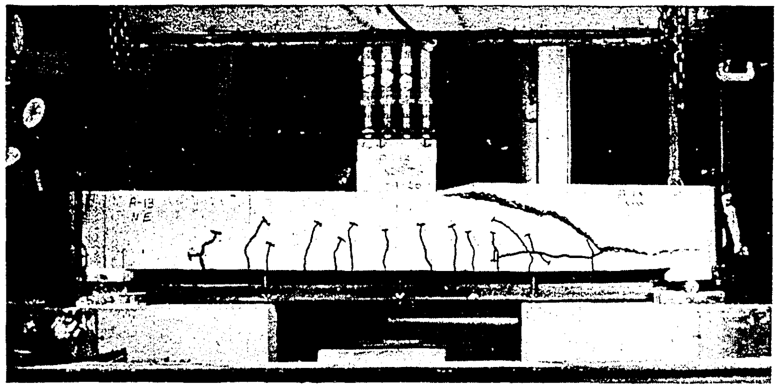


(a) Beam A-11 without axial load. $P_u = 46.5$ kips.

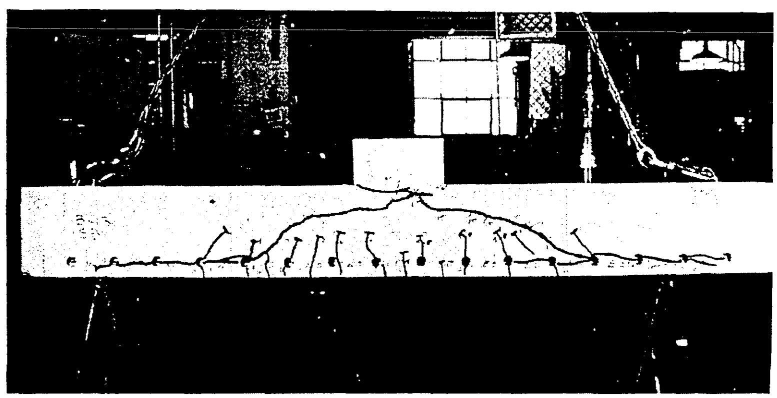


(b) Beam B-11 with axial load. $P_u = 64.5$ kips.

FIG. 13 EFFECT OF AXIAL LOAD ON SHEAR-COMPRESSION FAILURE



(a) Beam A-13 without axial load. P_u = 21.1 kips.



(b) Beam B-13 with axial load. P_u = 27.8 kips.

FIG. 14 EFFECT OF AXIAL LOAD ON MODE OF FAILURE

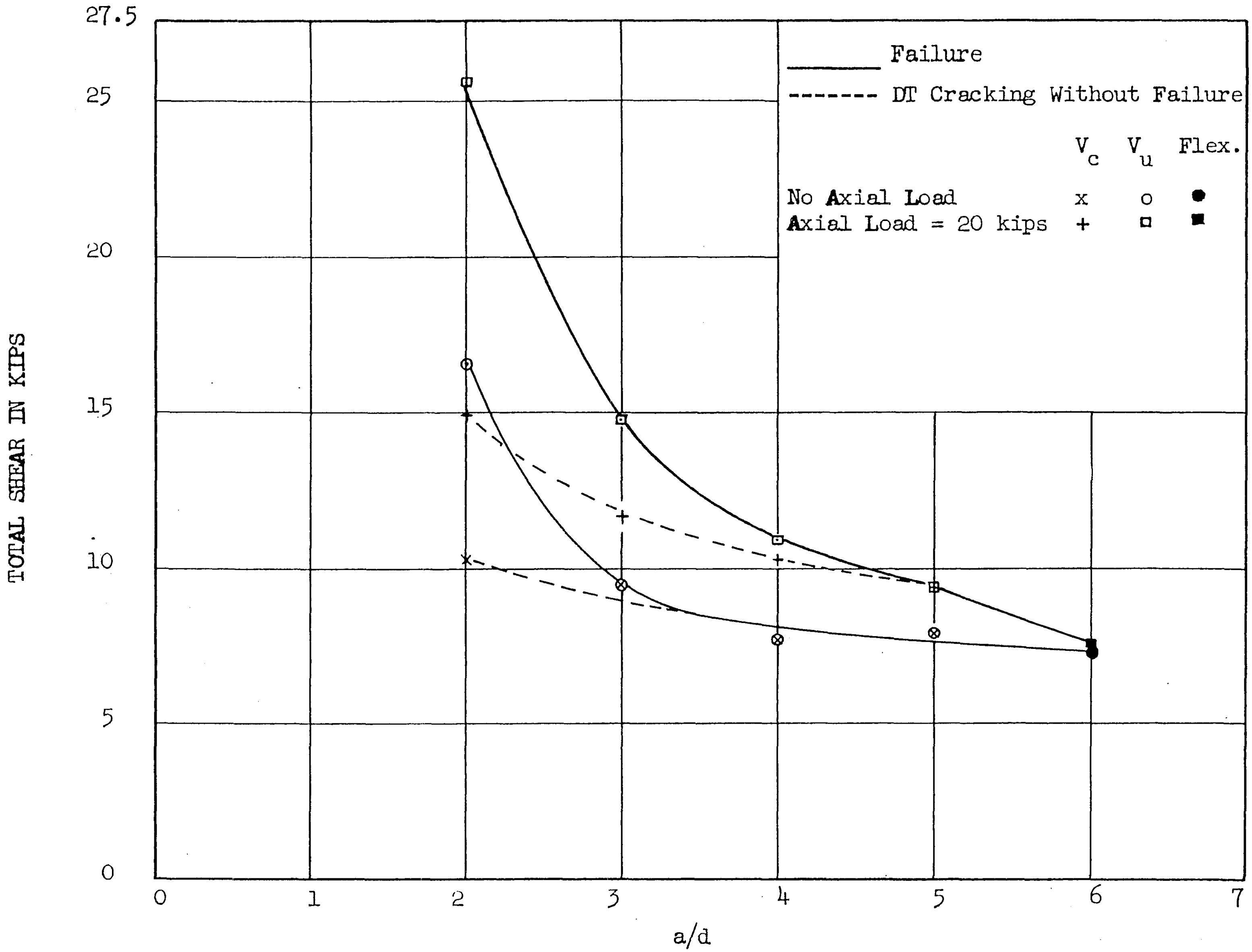


FIG. 15 EFFECT OF a/d ON CRACKING AND ULTIMATE SHEAR CAPACITY FOR BEAMS WITH THREE NO. 4 BARS

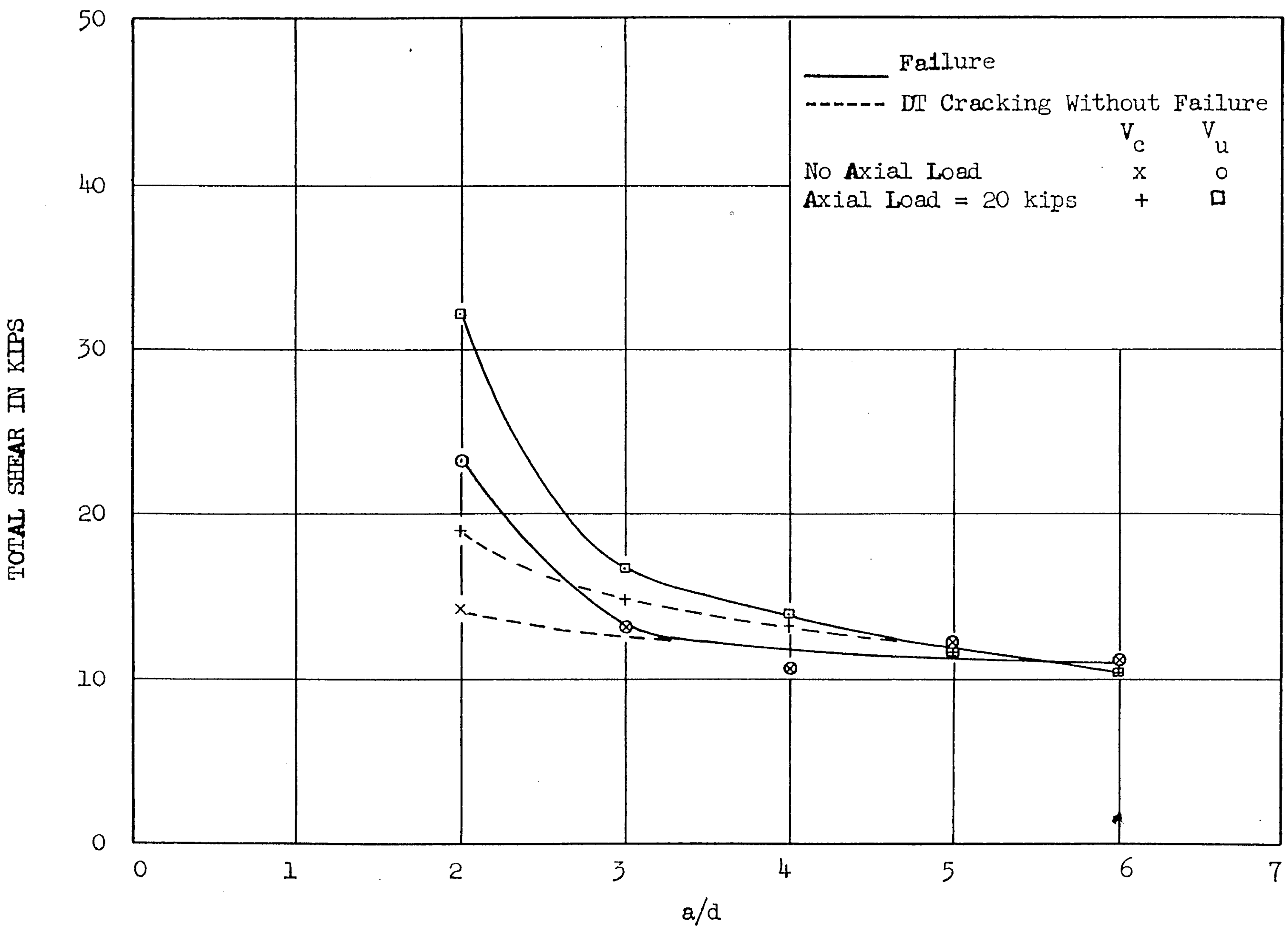


FIG. 16 EFFECT OF a/d ON CRACKING AND ULTIMATE SHEAR CAPACITY FOR BEAMS WITH TWO NO. 9 BARS

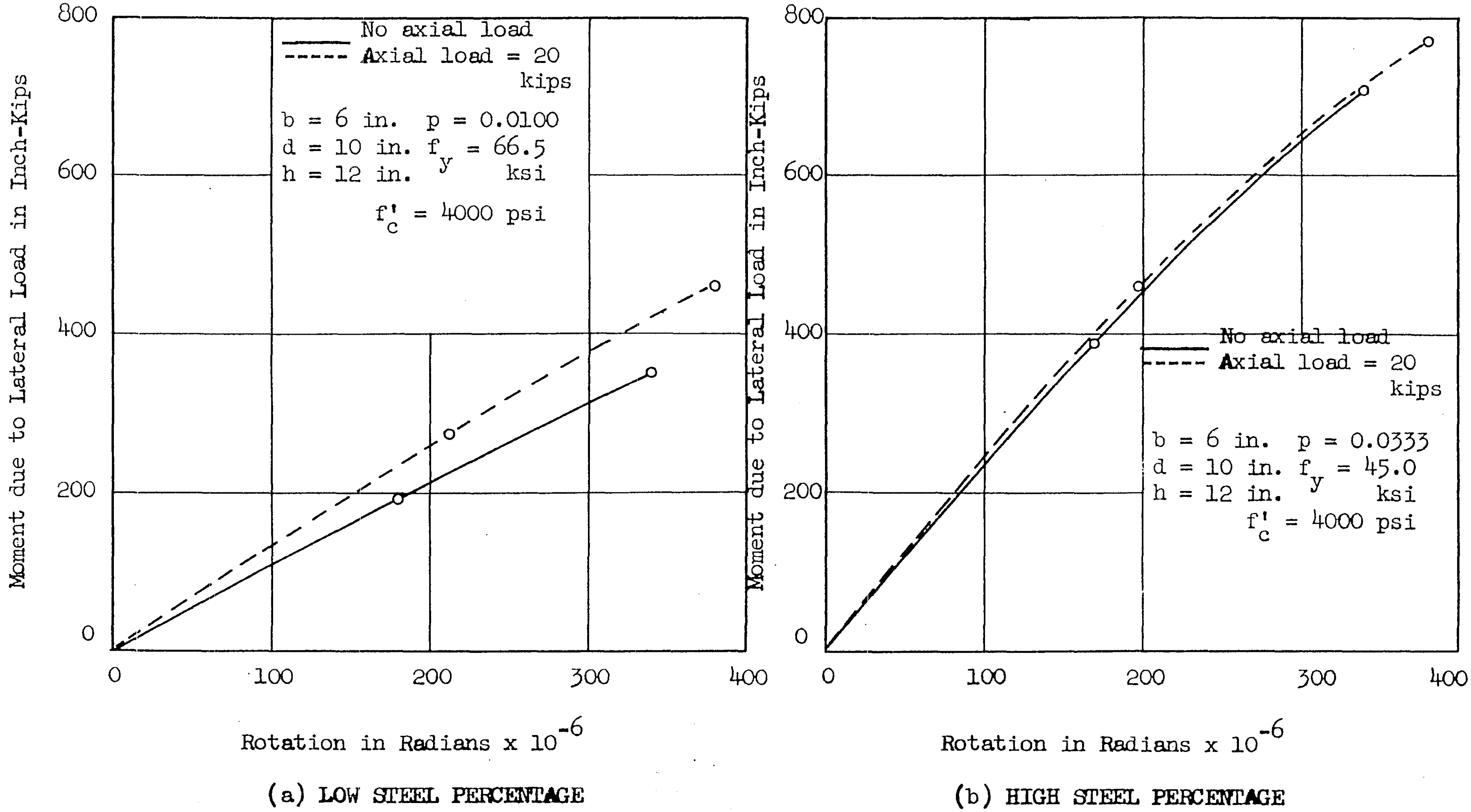


FIG. 17 EFFECT OF AXIAL LOAD ON MOMENT-ROTATION RELATIONSHIPS

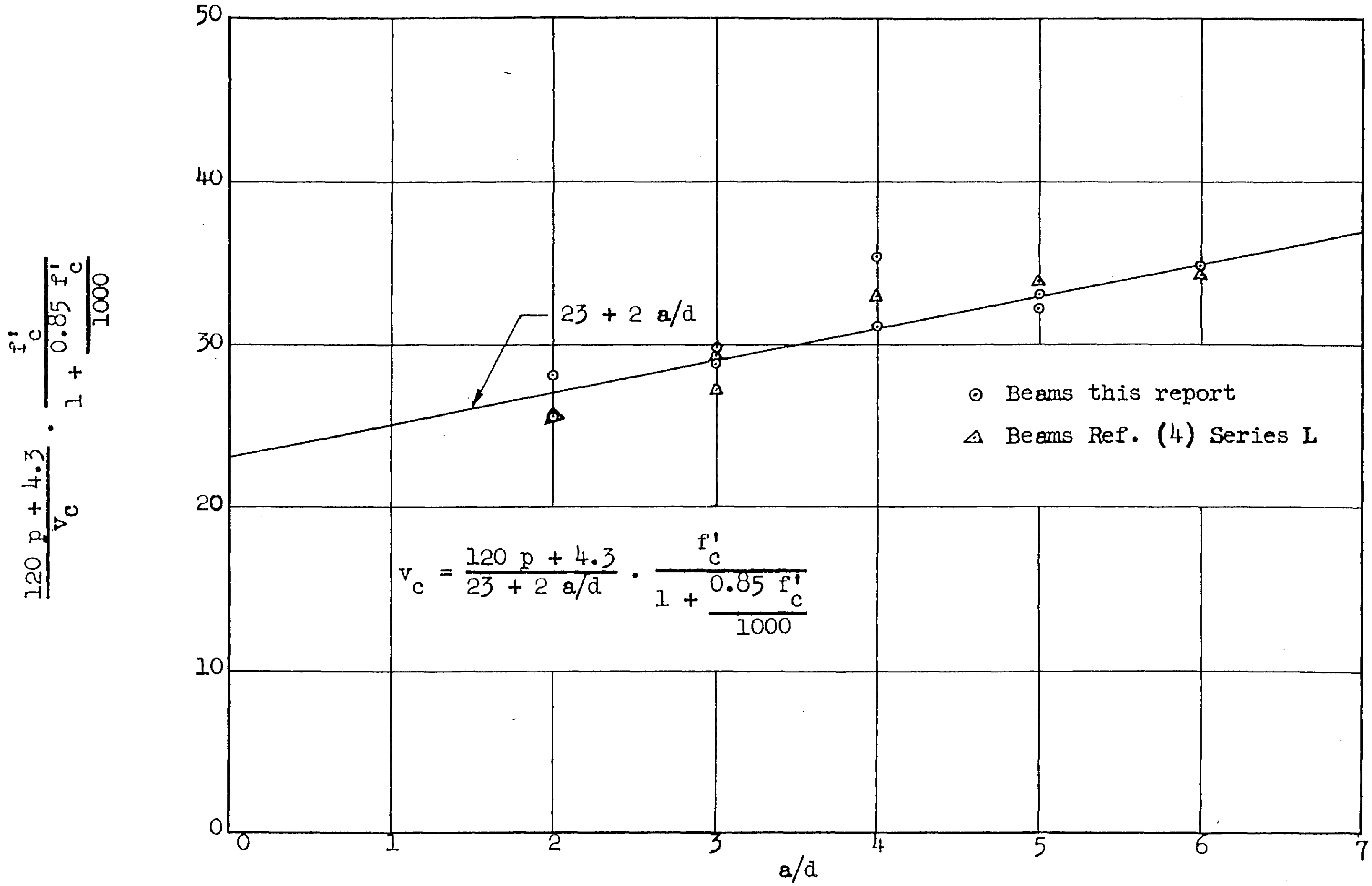


FIG. 18 **DIAGONAL TENSION CRACKING WITHOUT AXIAL LOAD**

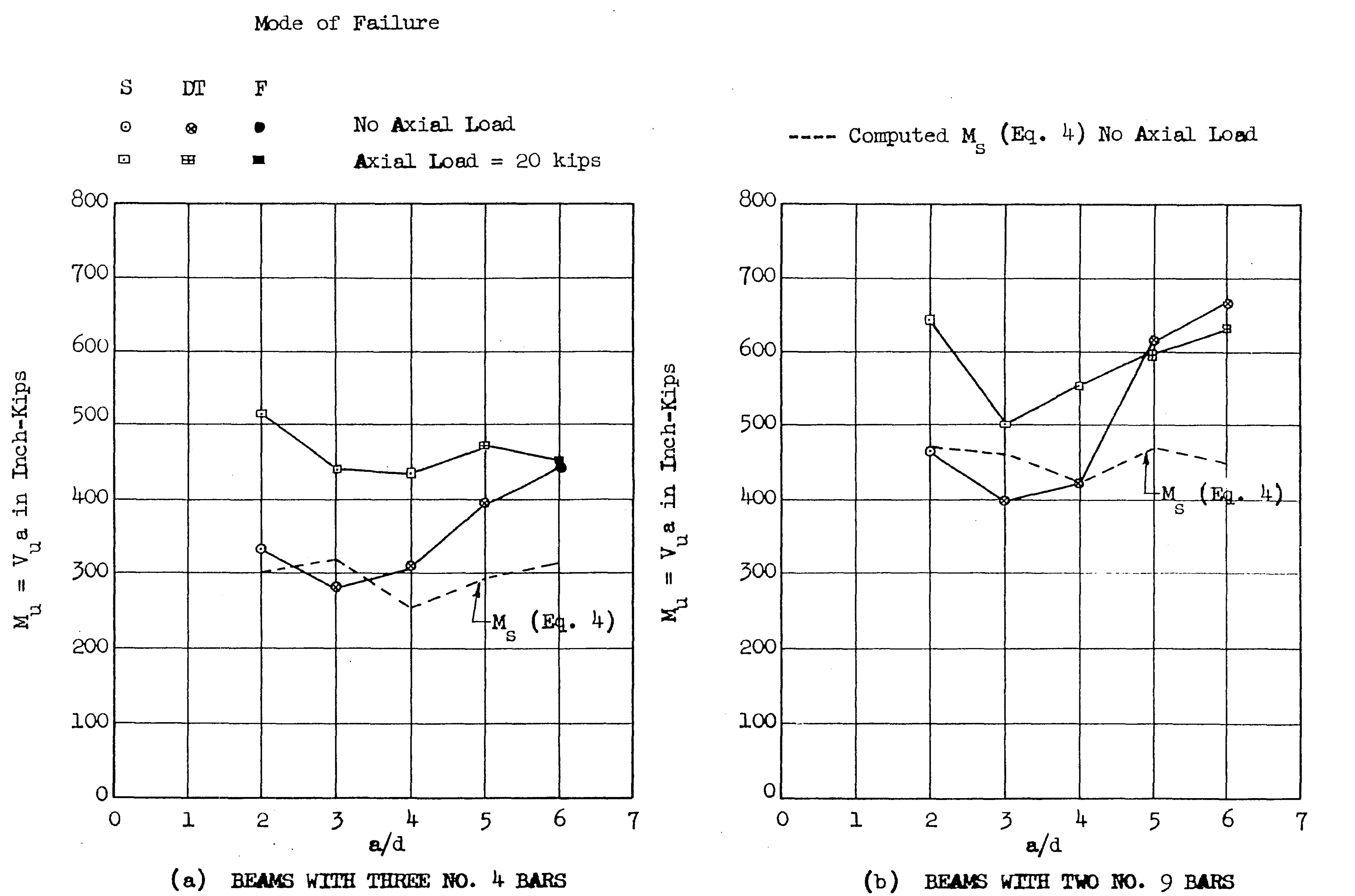


FIG. 19 ULTIMATE MOMENT VERSUS SHEAR-SPAN DEPTH RATIO

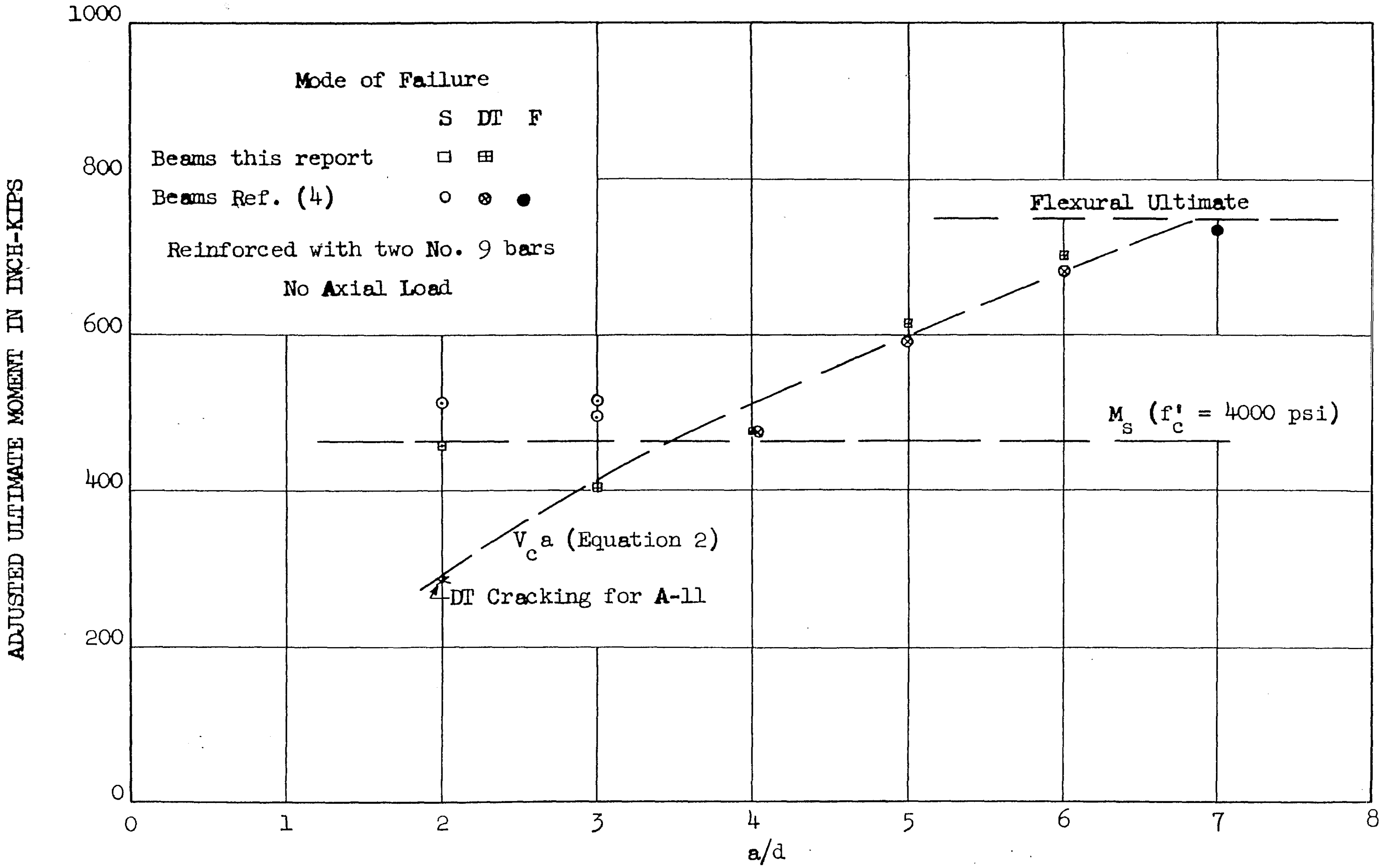


FIG. 20 COMPARISON WITH TEST RESULTS FOR BEAMS OF REFERENCE (4)